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ABSTRACT

Reported is an investigation designed to (1) utilize the Carroll model as a framework for implementing a mastery learning strategy in a nongraded setting, (2) identify the Carroll model as a possible theoretical basis for administrative decisions regarding the organizational structuring of schools, and (3) test certain hypotheses derived from the model which have implications concerning school organization. The Carroll model of school learning is a paradigm describing the degree of learning that occurs in a school setting as a function of time spent on a learning task divided by time needed for its mastery. Seventeen null hypotheses were investigated in the study which used a sample of 169 students enrolled in an algebra I unit focusing upon the four basic operations. Among the major findings was that a high quality of instruction characterized by feedback/correction procedures fostered a significantly greater degree of learning among students and a significantly greater number of classes spent by students than did a low quality of instruction characterized by the absence of feedback/correction procedures. The Carroll model's hypothesized interaction between ability to understand instruction and quality of instruction relative to degree of learning was confirmed statistically. (Author/PEB)

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FINAL REPORT

AN INVESTIGATION OF JOHN B. CARROLL'S MODEL OF SCHOOL LEARNING AS A THEORETICAL BASIS FOR THE ORGANIZATIONAL STRUCTURING OF SCHOOLS

National Institute of Education Project No. 3-1359

Dr. Glenn M. Hymel

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New Orleans, Louisiana

1974

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ABSTRACT

The Carroll Model of School Learning

The Carroll model of school learning is a paradigm describing the degree of learning that occurs in a school setting as a function of time spent on a learning task divided by time needed for its mastery. Five variables comprise the model: opportunity, perseverance, aptitude, ability to understand instruction, and quality of instruction. Opportunity and perseverance determine time spent while aptitude, ability to understand instruction, and quality of instruction determine time needed.

Purpose

Past research concerning the vertical structuring of schools has endeavored to demonstrate the superiority of nongradedness over gradedness as the more viable organizational approach for accommodating the individual differences among students. No attempts have been made, however, to examine critically the theoretical basis for school organization. The objectives of this investigation, therefore, encompassed the following:

1. To utilize the Carroll model as a framework for implementing a mastery learning strategy in a nongraded setting.

2. To identify the Carroll model as a possible theoretical basis for administrative decisions regarding the organizational structuring of schools.

3. To test certain hypotheses derived from the model which have implications concerning school organization.

Methodology and Procedures

Seventeen null hypotheses were investigated relative to the following three dependent variables: (_a_) degree of learning, (_b_) classes spent, and (_c_) perseverance. The various null hypotheses pertained to the following three areas: (_a_) main effects of quality of instruction relative to degree of learning, classes spent, and perseverance; (_b_) interaction effects of ability to understand instruction and quality of instruction relative to degree of learning, classes spent, and perseverance; and (_c_) correlations between and among degree of learning, classes spent, perseverance, and ability to understand instruction.

The sample was identical to a population of 169 students enrolled in an Algebra I unit which focused upon the four basic operations. This sample was divided into three ability levels based upon intelligence quotient scores. Students within each ability level were randomly assigned to two levels of treatment. The mastery learning group was exposed to a high quality of instruction characterized by performance objectives and feedback/correction procedures.

The control group was exposed to a low quality of instruction characterized only by performance objectives.

The experimental design used was a logical extension of the Posttest-Only Control Group Design. The crossing of three ability levels with two treatment levels resulted in a 3 X 2 fixed-effects factorial design. The major statistical techniques used included the following: analysis of variance, analysis of covariance, and Pearson product-moment correlation.

Findings

1. Main effects of treatment relative to degree of learning and classes spent were significant at the .001 level and favored the mastery learning group.

2. Main effects of treatment relative to perseverance were not significant.

3. Interaction effects of ability to understand instruction and quality of instruction relative to degree of learning were significant at the .05 level and favored Carroll's hypothesized interaction.

4. Interaction effects of ability to understand instruction and quality of instruction relative to classes spent and perseverance were not significant.

5. The mastery learning and control groups' correlation coefficients for perseverance and ability to understand instruction were significantly different at the .05 level.

6. The mastery learning and control groups' correlation coefficients for various other combinations of variables were not significant.

Conclusions

1. A high quality of instruction characterized by feedback/correction procedures fostered a significantly greater degree of learning among students and a significantly greater number of classes spent by students than did a low quality of instruction characterized by the absence of feedback/correction procedures.

2. Carroll's hypothesized interaction between ability to understand instruction and quality of instruction relative to degree of learning was confirmed statistically.

3. The mastery learning and control groups' correlation coefficients for perseverance and ability to understand instruction were differentially affected to a significant degree by the treatments.

CHAPTER 1

Introduction

It has been observed by Goodlad (1962) that ". . . school organization profoundly influences the answers which will be given to classic educational questions such as who shall be educated, what shall be taught, and when specified learnings shall be introduced. Determining school structure appropriate to the educational processes deemed desirable is a pressing problem (p. 210)." One aspect of this issue which has received an abundance of attention concerns the vertical pattern of school organization, that is, the organizational structure of a school which dictates the manner in which students progress upward along the various curricular sequences from year to year.

Alternative Forms of School Organization

Reflecting upon the organizational structuring of schools, Bloom (1968) noted that the vast majority of educational institutions throughout the world are organized on a graded basis, that is, organized to provide group instruction with specified and limited periods of time allowed for the mastery of a given learning task. It is his position that "whatever the amount of time allowed by the school and the curriculum for particular subjects or learning tasks, it is likely to be too much for some students and not enough for other students (p. 7)."

In recognition of the apparent inability of the graded form of organization to accommodate the individual differences among students, the nongraded approach to school organization has been suggested as a viable alternative. Smith (1970) described nongradedness as providing for the ". . . continuous progression of all students, with recognition of the variability among them in all aspects of their development. This type of school organization provides for differentiated rates and means of progression toward achievement of educational goals (p. 21)." Furthermore, Franklin (1967) asserted that nongradedness is ". . . primarily an administrative arrangement that recognizes individual differences; meets a student where he is; provides for his continuous progress at his own speed; . . . discards grade labels (i.e., first, second, third, etc.); replaces grade standards and uniform academic requirements with sequential subject-matter levels; . . . (p. 331)."

In essence, then, the nongraded form of organization purports to accommodate the varying academic needs and learning rates among students by way of making feasible the progression of a student through the contents of a given subject independent of any constant time boundaries. In other words, the subject matter is the constant while the time allowed for the mastery of the topics is the variable. Due to this unrestricted variability of time, the administrative structure of the school allows for the eventuality that a student may complete a given course irrespective of his grade level, chronological age, and/or month of the school year.

Past Approaches Taken in Research on School Organizational Structuring

The theoretical rationale which has been identified as the basis for nongradedness is the position that individual learners differ with respect to their potentialities for achievement and interest in various subject areas and, therefore, must be permitted to operate under a form of school organization which is amenable--and indeed conducive--to each student progressing at a rate dictated by his own capabilities.

As a result of this theoretical orientation, past research concerning the vertical structuring of schools has attempted to demonstrate the superiority of nongradedness over gradedness as the more viable organizational approach for accommodating the individual differences among students. After having examined 41 research reports which endeavored to evaluate nongraded programs, Otto (1971) asserted that the results are contradictory and inconclusive. Though not included in Otto's review, similar results of a contradictory and inconclusive nature were found in and/or among individual studies by Bowman (1971), Hunt (1970), Jackson (1965), Killough (1971), Ramayya (1971), Remacle (1970), Steere (1968), and Zerby (1960).

It is the contention of this writer that what is needed in the way of research concerning the organizational structuring of schools is not just simply a comparison of nongradedness with gradedness on various dependent variables. Although this methodological approach undoubtedly has some merit by virtue of its comparative nature, it appears that a research strategy which looks directly at the theoretical basis for school organization is in dire need.

Purpose of the Study

The literature currently available on alternative forms of school organization can be characterized as barren with respect to the identification and investigation of a specific model which might serve as a theoretical justification for the manner in which schools are organized. Though not specifically presented as a conceptual paradigm underlying school organizational patterning, John B. Carroll's (1963) model of school learning does represent at least a potentially tenable basis for decisions in this area of administration.

With this in mind, then, the objectives of this study included the following:

1. To utilize the Carroll model of school learning as a framework for implementing a mastery learning strategy in a nongraded setting.

2. To identify the Carroll model as a possible theoretical basis for administrative decisions regarding the organizational structuring of schools.

3. To test certain hypotheses derived from the model which have implications concerning school organization.

The Carroll Model of School Learning

Overview of the Model

According to Carroll (1963), there does exist a definite need for ". . . a schematic design or conceptual model of factors affecting success in school learning and of the way they interact (p. 723)." Working from a recognition of this need, Carroll has formulated a model of school learning which asserts that the success a student achieves in mastering a given learning task is contingent upon the extent to which he spends the amount of time that he needs to learn the task. More specifically, the Carroll model is a theoretical paradigm which maintains that in a school setting degree of learning is a function of the time spent on a learning task divided by the time needed for its mastery. The basic formulation of the model is expressed as follows:

$$\text{Degree of Learning} = f\,\frac{(\text{Time Spent})}{(\text{Time Needed})}$$

Encompassed within Carroll's model is a total of five factors or variables which independently and through interactions serve to determine the degree of learning that takes place with respect to a learning task. Three of the hypothesized variables determine how much time a student needs to spend in order to master a learning task; the remaining two variables determine the amount of time a student spends actively engaged in learning.

Components and Operations of the Model

The five factors or variables which comprise the Carroll model include the following: aptitude, ability to understand instruction, quality of instruction, opportunity, and perseverance. While aptitude, ability to understand instruction, and quality of instruction function as determinants of time needed, opportunity and perseverance serve as determinants of time spent. Each of these five components, as well as the manner in which they operate to determine the denominator (time needed) and the numerator (time spent) of the model, has been defined and explicated by Carroll (1963, 1965, 1970) in the following manner:

Aptitude. As one of the three determinants of time needed for learning, aptitude pertains to the amount of time required by a student to master a given task under optimal instructional conditions. The optimal conditions of instruction referred to include the following: (a) the implementation of those teaching techniques deemed most appropriate for a given student's own learning needs, (b) the provision of an ample amount of time during which the student could master the task, and (c) a willingness on the part of the student to spend the necessary time required to attain mastery. As viewed from the perspective of the model, aptitude refers to potential learning rate rather than potential level or complexity of learning and, hence, differs from the conventional psychometric connotations associated with the expression.

Ability to understand instruction. Proposed as a variable independent of aptitude, but contributing to the determination of time needed nonetheless, ability to understand instruction refers to the ability of a student to perceive and to understand (a) the nature of the learning task with which he is confronted and (b) the procedures he must follow in order to master the task. This component of the model can be thought of as the generalized ability of a student to benefit from the explanations of teachers and instructional materials. Due to the generalized nature of this variable as well as the highly verbal orientation of our schools, appropriate indices of ability to understand instruction include measures of general intelligence, verbal ability, reading comprehension, listening skills, or some combination thereof.

Quality of instruction. The third determinant of time needed, quality of instruction, is defined as the extent to which the various elements of a learning task are organized, presented, and explained in a manner commensurate with the special needs and characteristics of the learner. In elaborating upon this variable, Carroll (1970) asserted that the following three items are of particular importance in the endeavor to provide a high quality of instruction: (a) the specification of objectives for the purpose of identifying exactly what the learning task is, (b) the appropriate sequencing of learning subtasks, and (c) the employment of formative or diagnostic tests along with the accompanying prescription of alternative resources for remedial and/or enrichment activities.

Carroll's hypothesized interaction between ability to understand instruction and quality of instruction. Two of the determinants of time needed for learning, ability to understand instruction and quality of instruction, are hypothesized to interact in such a way that students low in ability to understand instruction will suffer more with respect to degree of learning when subjected to low quality of instruction than will students high in ability to understand instruction. It is reasoned that

> learners with high ability . . . . will be able to figure out for themselves what the learning task is and how they can go about learning it; they will be able to overcome the difficulties presented by poor quality of instruction by perceiving concepts and relationships in the teaching materials which will not be grasped by those with lesser ability (1963, p. 727).

The determination of time needed for learning. In those situations where quality of instruction is less than optimal for a given learner, additional time will be needed beyond that already required by virtue of his aptitude for the particular task being confronted. Furthermore, the amount of extra time required will be inversely related to the student's ability to understand instruction.

Thus, the actual time needed by a student for the mastery of a given learning task can be expressed as a function of his aptitude (amount of time required for learning under optimal instructional conditions) plus the interaction between ability to understand instruction and quality of instruction (additional time needed for learning as determined by the student's level of ability to understand instruction and the extent to which the quality of instruction deviates from that which is optimal for the learner).

Opportunity. Opportunity, the first of two determinants of time spent, refers to the total amount of time allowed or made available for learning. The allotment of a given amount of time for learning which is less than either the time needed by the student or the time he is willing to spend serves to reduce the degree of learning and, thus, results in incomplete learning. This type of situation which involves inadequate time allotments is especially detrimental to the student who is confronted with a poor quality of instruction and/or possesses a low ability to understand instruction.

Perseverance. As the second factor of the model which determines time spent in learning, perseverance is defined as the amount of time a student is willing to spend actively engaged in a learning task. In those instances where the perseverance of a student is restricted due to insufficient time allotments (opportunity) or is less than the time needed by the student, the degree of learning that results is less than optimal. Though not explicitly stated in the model, it can be inferred logically that ability to understand instruction and quality of instruction do exert an interactive effect upon perseverance in a manner similar to their hypothesized interactive effect upon degree of learning.

The determination of time spent in learning. The time actually spent in learning by a student is viewed as being equal to the smallest of the following three factors: (a) opportunity, (b) perseverance, and (c) time needed. Concerning the rationale behind the use of time needed as a potential determiner of time spent, a basic assumption in the model is that a student will never spend more time than he actually needs to master a given task. Hence, when the time allowed (opportunity) for learning permits the student to persevere to mastery, time spent is then equal to time needed.

Summary of the Model

The five factors of the model, aptitude, ability to understand instruction, quality of instruction, opportunity, and perseverance, have been placed by Carroll (1963) into a tentative formula which asserts that the degree of learning achieved by the ith individual with respect to the tth task is a function of the ratio of the time actually spent in learning to the time needed for learning. Hence,

$$\text{Degree of Learning} = f\,\frac{(\text{Time Spent})}{(\text{Time Needed})}$$

The time actually spent in learning is always equal to the smallest of the following three quantities: (a) opportunity, (b) perseverance, and (c) time needed. The time needed for learning is always equal to the aptitude which a student has for a particular learning task, increased by whatever additional time is required as a result of the interaction between ability to understand instruction and the quality of instruction when the latter is less than optimal. When the time spent by the student on the learning task is identical to the time needed to learn the task, the degree of learning will be 1.00, thus implying total mastery.

Concluding Observations Relative to the Model

Having examined the various components and operations of the model, it is appropriate at this point to acknowledge certain observations forwarded by Carroll (1963) which may serve to clarify the intent of the model as well as the parameters under which it functions. These include the following:

1. The reality of the learning process is an a priori assumption of the model. Consequently, the model of school learning should not be thought of as a "learning theory" which analyzes the necessary conditions for learning and tries to formulate a systematic explanation of this phenomenon. Instead, the model simply purports to contain, directly or indirectly, every element which influences a student's success or failure in school learning.

2. Carroll's model endeavors to provide a mathematical description of how various factors influence the degree to which a learning task is mastered. Within the context of the model, a learning task represents the student's ". . . going from ignorance of some specified fact or concept to knowledge or understanding of it, or of proceeding from

incapability of performing some specified act to capability of performing it . . . (p. 723)." In order for a learning task to be applicable to the model, it is mandatory that two conditions be met: (a) the task must be unequivocally described and (b) appropriate means must be specified which will provide a valid indication of when the task has been accomplished satisfactorily.

3. The model is not applicable to those goals of a school which are of an affective nature. Although learning tasks may very well contribute to the cognitive support of certain attitudes and dispositions deemed desirable by the school, it is assumed that the acquisition of affective behaviors adheres to a theoretical paradigm which differs from that involved in learning tasks.

4. Although the model is formulated in terms of the degree of learning attained on one learning task, ". . . it should be possible in principle to describe the pupil's success in learning a series of tasks . . . by summating the results of applying the model successively to each component task (p. 724)."

5. Any reference in the model to "time spent" is intended to mean "time actually spent" in learning. "'Time' is therefore not 'elapsed time' but the time during which the person is oriented to the learning task and actively engaged in learning (p. 725)."

Assumptions Pertinent to the Study

Certain basic assumptions explicitly stated or implicitly suggested by Carroll (1963) in his model of school learning have specific relevance to the hypotheses tested in this study. These assumptions include the following:

1. As quality of instruction increases, there is a corresponding decrease in the time needed by a student for learning, thus resulting in a closer approximation of 1.00 degree of learning, or mastery learning. The general assumption, then, is that high quality of instruction implies a greater degree of learning than does low quality of instruction.

2. Success in learning serves to increase one's willingness to persevere on a learning task more than does failure in learning. This assumption is related to the role of positive reinforcement in the context of operant conditioning as developed by Skinner (1954, 1968, 1971).

3. High quality of instruction implies a greater degree of learning than does low quality of instruction. Success in learning can be viewed as a positive reinforcer of one's willingness to persevere on a learning task. Therefore, it can be concluded logically that high quality of instruction implies a greater amount of perseverance on a learning task than does low quality of instruction.

4. Low ability to understand instruction implies a lesser degree of learning than does high ability to understand instruction. Low quality of instruction implies a

lesser degree of learning than does high quality of instruction. As explicitly hypothesized by Carroll, then, the interaction between ability to understand instruction and quality of instruction relative to degree of learning is such that students low in ability to understand instruction will suffer more when subjected to low quality of instruction than will students high in ability to understand instruction.

5. Low ability to understand instruction implies a greater need for perseverance on a learning task than does high ability to understand instruction. High quality of instruction implies a greater amount of perseverance on a learning task than does low quality of instruction. Therefore, though not explicitly stated in the model, it can be logically inferred that the interaction between ability to understand instruction and quality of instruction relative to perseverance on a learning task is such that students low in ability to understand instruction will persevere more when subjected to high quality of instruction and less when subjected to low quality of instruction than will students high in ability to understand instruction.

6. Low ability to understand instruction implies a lesser degree of learning than does high ability to understand instruction. High quality of instruction implies a greater degree of learning than does low quality of instruction. Therefore, degree of learning is less a direct function of ability to understand instruction under high quality of instruction than under low quality of instruction.

7. Low ability to understand instruction implies a greater need for perseverance on learning tasks than does high ability to understand instruction. High quality of instruction implies a greater amount of perseverance on a learning task than does low quality of instruction. Therefore, perseverance on a learning task is inversely related to ability to understand instruction under high quality of instruction but directly related to ability to understand instruction under low quality of instruction.

8. Carroll has implicitly suggested that, with all other variables in the model held constant, degree of learning is a function of the amount of perseverance on a learning task.

Definition of Terms

In addition to the terms which have already been defined, the following definitions are listed for the purpose of ensuring the proper interpretation of the terminology used in this study:

1. Mastery learning refers to a student's attainment of a prespecified degree of proficiency with respect to a given learning task. In terms of Carroll's model, mastery learning is in evidence when the ratio of time actually spent to time needed is equal to 1.00.

2. Mastery learning strategy refers to an instructional approach which accommodates the individual differences among students in such a way that the vast majority of the

students attain mastery of the learning task or tasks under consideration.

3. Formative evaluation refers to the continual assessment of a student's progress at various intermittent stages prior to the completion of a unit of instruction (Scriven, 1967). In this study the purpose of the formative evaluation procedures was to provide both the student and the teacher with on-going feedback relative to learning deficiencies experienced by the student as well as the alterations most needed in the instructional materials and strategies. The evaluative instruments used to accomplish this objective are labeled "formative trial tests."

4. The expression learning correctives refers to prescriptive exercises of a review and/or remedial nature assigned for the purpose of correcting any learning deficiencies experienced by the student as identified by the formative trial tests.

5. Mastery learning group (MLG) refers to those students in the sample who were exposed to a mastery learning strategy which included (a) the specification of performance objectives, (b) the use of formative trial tests, and (c) the prescription of learning correctives of a review and/or remedial nature. From the perspective of Carroll's model, this instructional strategy can be viewed as comprising a high quality of instruction.

6. Control group (CG) refers to those students in the sample who were exposed to an instructional strategy which

employed the specification of performance objectives but did not include any procedures of a feedback/correction nature. From the perspective of Carroll's model, this pedagogical approach can be viewed as comprising a low quality of instruction.

7. Summative evaluation refers to the final assessment of a student's total accomplishments at the conclusion of a unit of instruction (Scriven, 1967). In this study the purpose of the summative evaluation procedures was to assign numerical grades to students as an indication of the percentage of learning material mastered. The evaluative instruments used to accomplish this objective are labeled "summative posttests."

8. Degree of learning refers to the percentage of learning material mastered by each student and reported in the form of an achievement raw score on a summative posttest covering algebraic topics. This measure represents one of the three dependent variables investigated in this study.

9. Perseverance refers to the total number of minutes and seconds spent by each student on a difficult learning task administered subsequent to the summative posttest. As one of the three dependent variables investigated in the study, this highly controlled measure is consistent with Carroll's definition of perseverance as the amount of time a student is willing to spend actively engaged in a learning task. The evaluative instrument used to obtain this measure is labeled "assessment of perseverance."

10. The expression classes spent refers to the total number of instructional periods in an algebra course attended by each student while completing the unit of instruction used in this study. As one of the three dependent variables investigated in the study, it is acknowledged that this measure represents only an approximation of the highly controlled variable of perseverance identified in the Carroll model.

11. Ability to understand instruction refers to the total intelligence quotient score attained by each student on the California Short-Form Test of Mental Maturity, 1963 Revision. This measure is consistent with Carroll's description of ability to understand instruction as a generalized ability inclusive of both general intelligence and verbal ability.

Statement of the Problem

The educational setting in which this study was implemented can be described in the following manner: The mastery learning group and the control group operated within a nongraded form of school organization which provided students with unlimited time opportunity for the attainment of a maximum degree of learning over a series of learning tasks. Hence, the school organizational pattern permitted the expenditure of whatever amount of time was needed by the students in their quest for total mastery learning.

Based upon the assumptions that were identified earlier, this investigation endeavored to answer the following major question: Given a nongraded school organizational structure that provides students with unlimited time allotments for the mastery of a series of learning tasks, what are the main and interactive effects of quality of instruction and ability to understand instruction relative to the degree of learning attained and the amount of perseverance manifested by students?

More specifically, this study attempted to answer the following questions:

1. How will quality of instruction affect degree of learning in a setting of unlimited opportunity?

2. What will be the interaction between ability to understand instruction and quality of instruction relative to degree of learning in a setting of unlimited opportunity?

3. What will be the correlation between degree of learning and ability to understand instruction in a setting of unlimited opportunity while under a high quality of instruction? while under a low quality of instruction?

4. With the effects due to a rough estimate of perseverance held constant, how will quality of instruction affect degree of learning in a setting of unlimited opportunity?

5. With the effects due to a rough estimate of perseverance held constant, what will be the interaction between ability to understand instruction and quality of instruction relative to degree of learning in a setting of unlimited opportunity?

6. With the effects due to a rough estimate of perseverance held constant, what will be the correlation between

degree of learning and ability to understand instruction in a setting of unlimited opportunity while under a high quality of instruction? while under a low quality of instruction?

7. How will quality of instruction affect a rough estimate of perseverance in a setting of unlimited opportunity?

8. What will be the interaction between ability to understand instruction and quality of instruction relative to a rough estimate of perseverance in a setting of unlimited opportunity?

9. What will be the correlation between a rough estimate of perseverance and ability to understand instruction in a setting of unlimited opportunity while under a high quality of instruction? while under a low quality of instruction?

10. What will be the correlation between degree of learning and a rough estimate of perseverance in a setting of unlimited opportunity while under a high quality of instruction? while under a low quality of instruction?

11. How will quality of instruction affect perseverance in a setting of unlimited opportunity?

12. What will be the interaction between ability to understand instruction and quality of instruction relative to perseverance in a setting of unlimited opportunity?

13. What will be the correlation between perseverance and ability to understand instruction in a setting of unlimited opportunity while under a high quality of instruction? while under a low quality of instruction?

14. With the effects due to degree of learning held constant, how will quality of instruction affect perseverance in a setting of unlimited opportunity?

15. With the effects due to degree of learning held constant, what will be the interaction between ability to understand instruction and quality of instruction relative to perseverance in a setting of unlimited opportunity?

16. With the effects due to degree of learning held constant, what will be the correlation between perseverance and ability to understand instruction in a setting of unlimited opportunity while under a high quality of instruction? while under a low quality of instruction?

17. What will be the correlation between degree of learning and perseverance in a setting of unlimited opportunity while under a high quality of instruction? while under a low quality of instruction?

Significance of the Study

John B. Carroll's model of school learning purports to contain, directly or indirectly, every element required to account for an individual's success or failure in school learning. More importantly, the specific components of opportunity and perseverance have particular relevance to a form of school organization such as nongradedness which has as its primary objective the provision of unlimited time opportunity during which a student can engage actively in a learning task until the time spent is commensurate with the time needed.

What is needed in the area of research on school organization, then, is an investigation of the degree of learning and the amount of perseverance that actually occur under varying conditions of quality of instruction and ability to understand instruction when unlimited time opportunity is made available to students. The collection and analysis of data which substantiate the aforementioned assumptions and the corresponding hypotheses which follow would indeed be indicative of the validity of Carroll's model as a viable theoretical paradigm for justifying the nongraded structuring of schools.

Research Hypotheses

Based upon the various components of Carroll's model and the previously cited assumptions, the following research hypotheses were identified as being of significant relevance to this study and, therefore, were tested:

Research hypothesis 1. There will be a significant difference between the mastery learning group and the control group relative to degree of learning. More specifically, the mastery learning group will attain a significantly greater degree of learning than will the control group.

Experimental consequence. The mastery learning group will achieve significantly higher on a summative posttest than will the control group.

Where MAc_{MLG} = the mean achievement score on a summative posttest for the mastery learning group

and

MAc_{CG} = the mean achievement score on a summative posttest for the control group,

it is predicted that:

$$MAc_{MLG} > MAc_{CG}$$

Research hypothesis 2. There will be a significant ordinal interaction between ability to understand instruction and quality of instruction relative to degree of learning. More specifically, as students decrease in ability to understand instruction, their degree of learning will decrease in both the mastery learning group and the control group; however, the extent of decrease will be significantly greater in the control group than in the mastery learning group.

Experimental consequence. In the control group achievement scores on a summative posttest will be more severely retarded for students low in ability to understand instruction than for students high in ability to understand instruction. (See Figure 1 for an illustration of the hypothesized ordinal interaction between ability to understand instruction and quality of instruction relative to summative posttest achievement scores.)

Where $MAc_{MLG,LAb}$, $MAc_{MLG,AAb}$, and $MAc_{MLG,HAb}$ = the mean achievement scores on a summative posttest for the mastery learning group which is divided into low-, average-, and high-ability levels, respectively,

and

$MAc_{CG,LAb}$, $MAc_{CG,AAb}$, and $MAc_{CG,HAb}$ = the mean achievement scores on a summative posttest for the control group which is divided into low-, average-, and high-ability levels, respectively,

it is predicted that:

$$MAc_{MLG,LAb} - MAc_{CG,LAb} > MAc_{MLG,AAb} - MAc_{CG,AAb} >$$

$$MAc_{MLG,HAb} - MAc_{CG,HAb}$$

Research hypothesis 3. There will be a significant difference between the mastery learning group and the control group relative to the correlation between degree of learning and ability to understand instruction. More specifically, the correlation between degree of learning and ability to understand instruction will not deviate significantly from zero in the mastery learning group but will be significantly positive in the control group, and these correlations will differ significantly from each other.

Experimental consequence. The relationship between achievement scores on a summative posttest and ability to understand instruction will be indiscernible in the mastery learning group but will be of a significantly direct nature in the control group, and these relationships will differ significantly from each other.

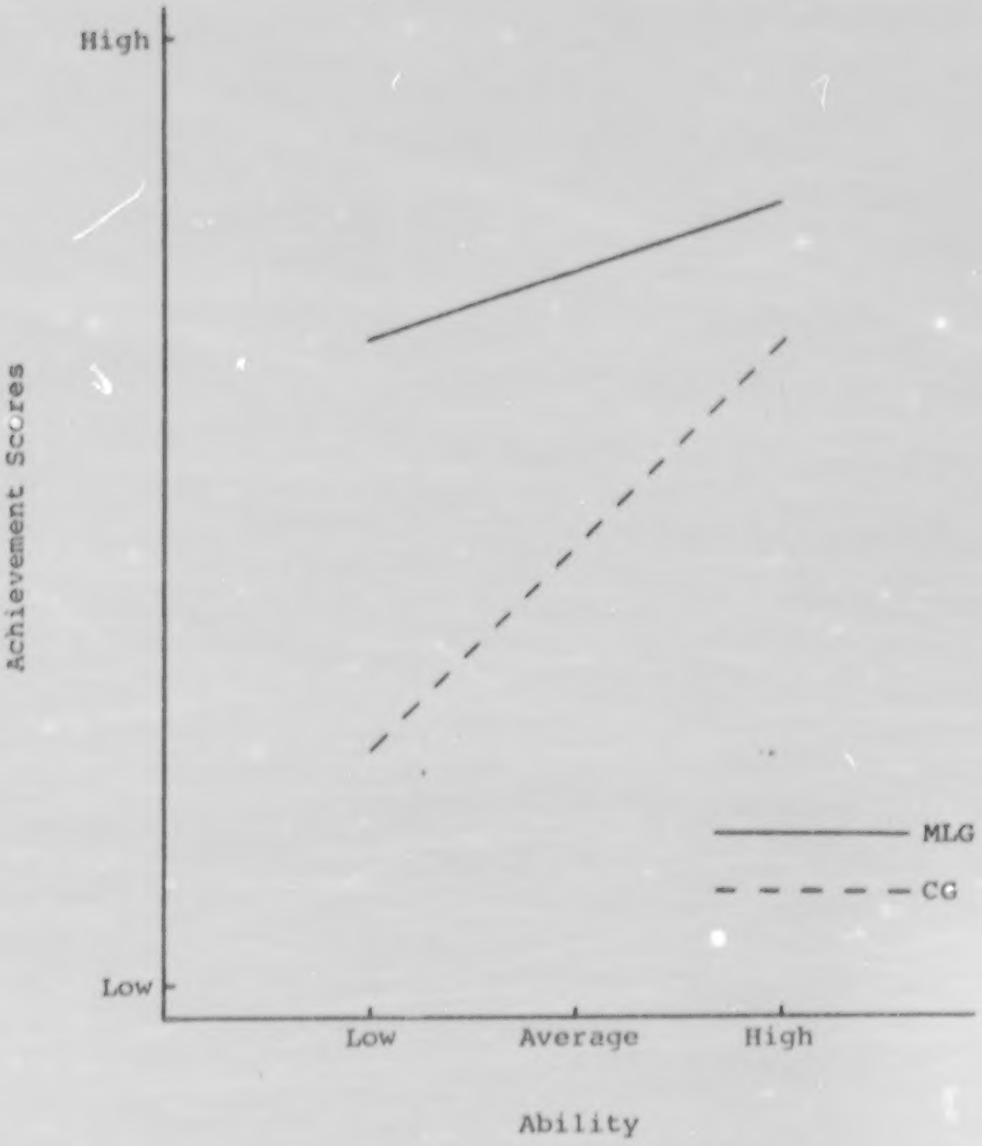

Fig. 1. Illustration of the hypothesized ordinal interaction between ability to understand instruction and quality of instruction relative to summative posttest achievement scores.

Where $r_{AcAb,MLG}$ = the correlation between achievement scores on a summative posttest and ability for the mastery learning group

and

$r_{AcAb,CG}$ = the correlation between achievement scores on a summative posttest and ability for the control group,

it is predicted that:

1. $r_{AcAb,MLG} \sim 0$
2. $r_{AcAb,CG} > 0$
3. $r_{AcAb,CG} - r_{AcAb,MLG} \neq 0$

Research hypothesis 4. With adjustments made for differences in classes spent, there will be a significant difference between the mastery learning group and the control group relative to degree of learning. More specifically, with adjustments made for differences in classes spent, the mastery learning group will attain a significantly greater degree of learning than will the control group.

Experimental consequence. With adjustments made for differences in the number of classes spent in completing a given unit of instruction, the mastery learning group will achieve significantly higher on a summative posttest than will the control group.

Where $MAc(adj.)_{MLG}$ = the mean achievement score on a summative posttest for the mastery learning group with adjustments made for differences in the number of classes spent in completing a given unit of instruction

and

$MAc(adj.)_{CG}$ = the mean achievement score on a summative posttest for the control group with adjustments made for differences in the number of classes spent in completing a given unit of instruction,

it is predicted that:

$$MAc(adj.)_{MLG} > MAc(adj.)_{CG}$$

Research hypothesis 5. With adjustments made for differences in classes spent, there will be a significant ordinal interaction between ability to understand instruction and quality of instruction relative to degree of learning. More specifically, with adjustments made for differences in classes spent, as students decrease in ability to understand instruction, their degree of learning will decrease in both the mastery learning group and the control group; however, the extent of decrease will be significantly greater in the control group than in the mastery learning group.

Experimental consequence. With adjustments made for differences in the number of classes spent in completing a unit of instruction, the control group's achievement scores on a summative posttest will be more severely retarded for students low in ability to understand instruction than for students high in ability to understand instruction. (See Figure 2 for an illustration of the hypothesized ordinal interaction between ability to understand instruction and quality of instruction relative to summative posttest achievement scores with adjustments made for differences in the number of classes spent.)

Where $MAc(adj.)_{MLG,LAb}$, $MAc(adj.)_{MLG,AAb}$, and $MAc(adj.)_{MLG,HAb}$ = the mean adhievement scores on a summative posttest for the mastery learning group which is divided into low-, average-, and high-ability levels, respectively, with adjustments made for differences in the number of classes spent in completing a given unit of instruction

and

$MAc(adj.)_{CG,LAb}$, $MAc(adj.)_{CG,AAb}$, and $MAc(adj.)_{CG,HAb}$ = the mean achievement scores on a summative posttest for the control group which is divided into low-, average-, and high-ability levels, respectively, with adjustments made for differences in the number of classes spent in completing a given unit of instruction,

it is predicted that:

$$MAc(adj.)_{MLG,LAb} - MAc(adj.)_{CG,LAb} > MAc(adj.)_{MLG,AAb} - MAc(adj.)_{CG,AAb} > MAc(adj.)_{MLG,HAb} - MAc(adj.)_{CG,HAb}$$

Research hypothesis 6. With the effects due to classes spent partialed out, there will be a significant difference between the mastery learning group and the control group relative to the correlation between degree of learning and ability to understand instruction. More specifically, with the effects due to classes spent partialed out, the correlation between degree of learning and ability to understand instruction will not deviate significantly from zero in the mastery learning group but will be significantly positive in the control group, and these correlations will differ significantly from each other.

Experimental consequence. With the effects due to the number of classes spent in completing a given unit of instruction partialed out, the relationship between achievement scores on a summative posttest and ability to understand

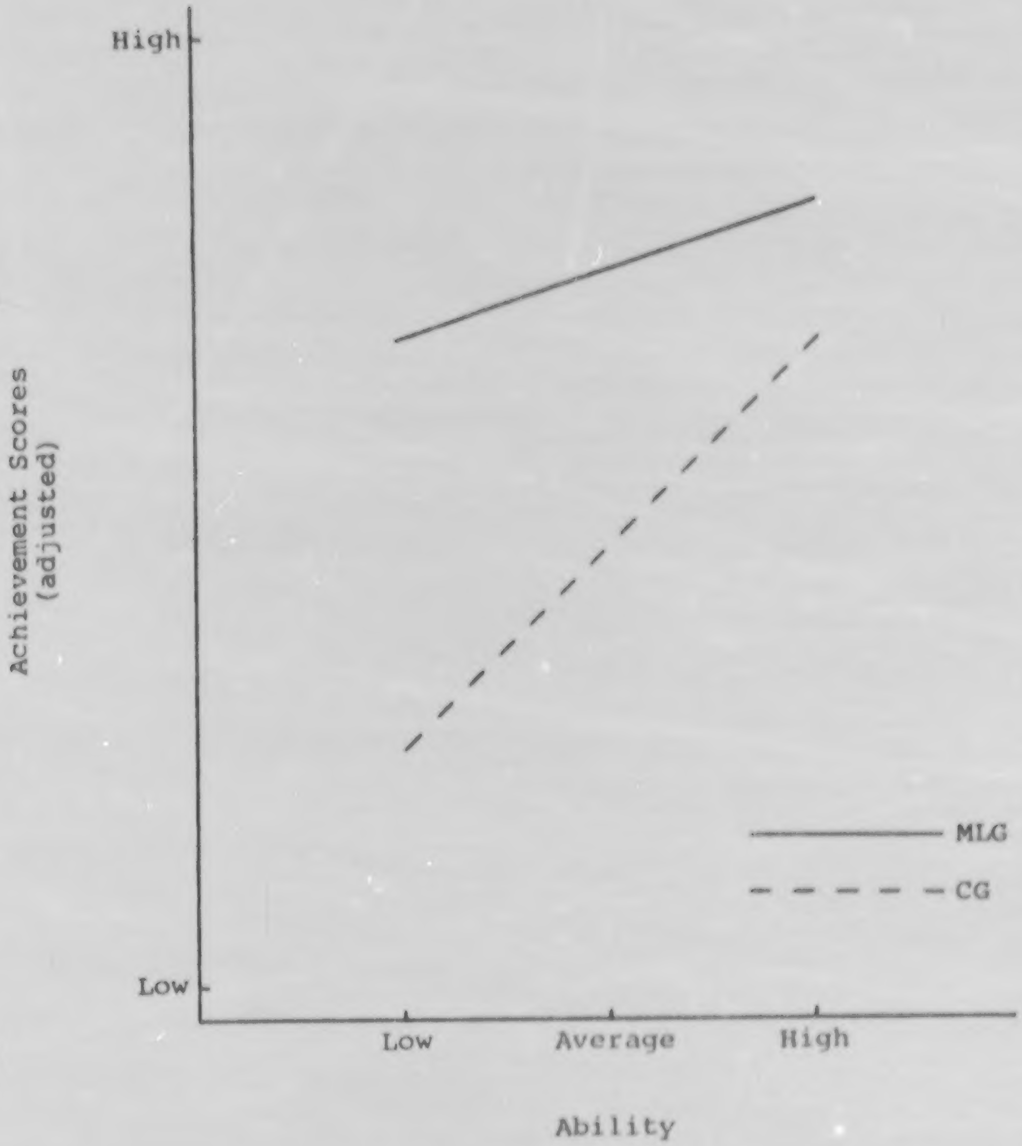

Fig. 2. Illustration of the hypothesized ordinal interaction between ability to understand instruction and quality of instruction relative to summative posttest achievement scores with adjustments made for differences in the number of classes spent.

instruction will be indiscernible in the mastery learning group but will be of a significantly direct nature in the control group, and these relationships will differ significantly from each other.

Where $r_{AcAb.CS,MLG}$ = the partial correlation between achievement scores on a summative posttest and ability with adjustments made for differences in the number of classes spent in completing a given unit of instruction for the mastery learning group

and

$r_{AcAb.CS,CG}$ = the partial correlation between achievement scores on a summative posttest and ability with adjustments made for differences in the number of classes spent in completing a given unit of instruction for the control group,

it is predicted that:

1. $r_{AcAb.CS,MLG} \sim 0$
2. $r_{AcAb.CS,CG} > 0$
3. $r_{AcAb.CS,CG} - r_{AcAb.CS,MLG} \neq 0$

Research hypothesis 7. There will be a significant difference between the mastery learning group and the control group relative to classes spent. More specifically, the mastery learning group will spend a significantly greater number of classes than will the control group.

Experimental consequence. The mastery learning group will spend a significantly greater number of classes in completing a given unit of instruction than will the control group.

Where MCS_{MLG} = the mean number of classes spent by the mastery learning group in completing a given unit of instruction

and

MCS_{CG} = the mean number of classes spent by the control group in completing a given unit of instruction,

it is predicted that:

$$MCS_{MLG} > MCS_{CG}$$

Research hypothesis 8. There will be a significant ordinal interaction between ability to understand instruction and quality of instruction relative to classes spent. More specifically, as students decrease in ability to understand instruction, the number of classes spent will increase in the mastery learning group but will decrease in the control group.

Experimental consequence. In the control group the number of classes spent in completing a given unit of instruction will be more severely limited for students low in ability to understand instruction than for students high in ability to understand instruction. (See Figure 3 for an illustration of the hypothesized ordinal interaction between ability to understand instruction and quality of instruction relative to the number of classes spent.)

Where $MCS_{MLG,LAb}$, $MCS_{MLG,AAb}$, and $MCS_{MLG,HAb}$ = the mean number of classes spent in completing a given unit of instruction by the mastery learning group which is divided into low-, average-, and high-ability levels, respectively,

and

$MCS_{CG,LAb}$, $MCS_{CG,AAb}$, and $MCS_{CG,HAb}$ = the mean number of classes spent in completing a given unit of instruction by the control group which is divided into low-, average-, and high-ability levels, respectively,

it is predicted that:

$$MCS_{MLG,LAb} - MCS_{CG,LAb} > MCS_{MLG,AAb} - MCS_{CG,AAb} >$$
$$MCS_{MLG,HAb} - MCS_{CG,HAb}$$

Research hypothesis 9. There will be a significant difference between the mastery learning group and the control group relative to the correlation between classes spent and ability to understand instruction. More specifically, the correlation between classes spent and ability to understand instruction will be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations will differ significantly from each other.

Experimental consequence. The relationship between the number of classes spent in completing a given unit of instruction and ability to understand instruction will be of a significantly inverse nature in the mastery learning group but of a significantly direct nature in the control group, and these relationships will differ significantly from each other.

Where $r_{CSAb,MLG}$ = the correlation between number of classes spent in completing a given unit of instruction and ability for the mastery learning group

and

$r_{CSAb,CG}$ = the correlation between number of classes spent in completing a given unit of instruction and ability for the control group,

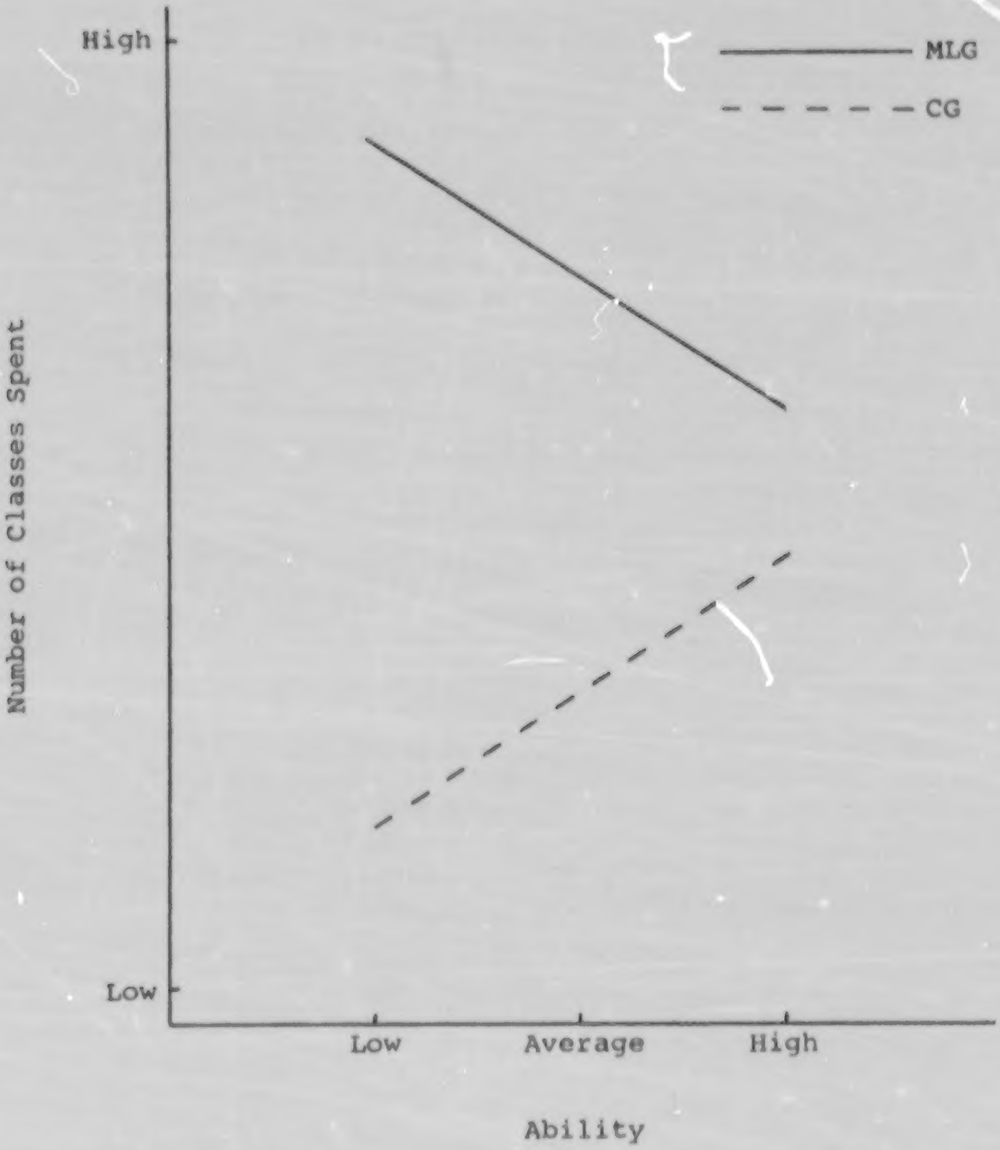

Fig. 3. Illustration of the hypothesized ordinal interaction between ability to understand instruction and quality of instruction relative to the number of classes spent.

it is predicted that:

1. $r_{CSAb,MLG} < 0$
2. $r_{CSAb,CG} > 0$
3. $r_{CSAb,CG} - r_{CSAb,MLG} \neq 0$

Research hypothesis 10. There will be a significant difference between the mastery learning group and the control group relative to the correlation between degree of learning and classes spent. More specifically, the correlation between degree of learning and classes spent will be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations will differ significantly from each other.

Experimental consequence. The relationship between achievement scores on a summative posttest and the number of classes spent in completing a given unit of instruction will be of a significantly inverse nature in the mastery learning group but of a significantly direct nature in the control group, and these relationships will differ significantly from each other.

Where $r_{AcCS,MLG}$ = the correlation between achievement scores on a summative posttest and number of classes spent in completing a given unit of instruction for the mastery learning group

and

$r_{AcCS,CG}$ = the correlation between achievement scores on a summative posttest and number of classes spent in completing a given unit of instruction for the control group,

it is predicted that:

1. $r_{AcCS,MLG} < 0$
2. $r_{AcCS,CG} > 0$
3. $r_{AcCS,CG} - r_{AcCS,MLG} \neq 0$

Research hypothesis 11. There will be a significant difference between the mastery learning group and the control group relative to perseverance. More specifically, the mastery learning group will manifest a significantly greater amount of perseverance than will the control group.

Experimental consequence. The mastery learning group will spend a significantly greater number of minutes in persevering on a difficult learning task than will the control group.

Where MP_{MLG} = the mean number of minutes spent by the mastery learning group in persevering on a difficult learning task

and

MP_{CG} = the mean number of minutes spent by the control group in persevering on a difficult learning task,

it is predicted that:

$$MP_{MLG} > MP_{CG}$$

Research hypothesis 12. There will be a significant ordinal interaction between ability to understand instruction and quality of instruction relative to perseverance. More specifically, as students decrease in ability to understand instruction, the amount of perseverance manifested will increase in the mastery learning group but will decrease in the control group.

Experimental consequence. In the control group the number of minutes spent in persevering on a difficult learning task will be more severely limited for students low in ability to understand instruction than for students high in ability to understand instruction. (See Figure 4 for an illustration of the hypothesized ordinal interaction between ability to understand instruction and quality of instruction relative to the number of minutes spent in persevering.)

Where $MP_{MLG,LAb}$, $MP_{MLG,AAb}$, and $MP_{MLG,HAb}$ = the mean number of minutes spent in persevering on a difficult learning task by the mastery learning group which is divided into low-, average-, and high-ability levels, respectively,

and

$MP_{CG,LAb}$, $MP_{CG,AAb}$, and $MP_{CG,HAb}$ = the mean number of minutes spent in persevering on a difficult learning task by the control group which is divided into low-, average-, and high-ability levels,

it is predicted that:

$$MP_{MLG,LAb} - MP_{CG,LAb} > MP_{MLG,AAb} - MP_{CG,AAb} >$$
$$MP_{MLG,HAb} - MP_{CG,HAb}$$

Research hypothesis 13. There will be a significant difference between the mastery learning group and the control group relative to the correlation between perseverance and ability to understand instruction. More specifically, the correlation between perseverance and ability to understand instruction will be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations will differ significantly from each other.

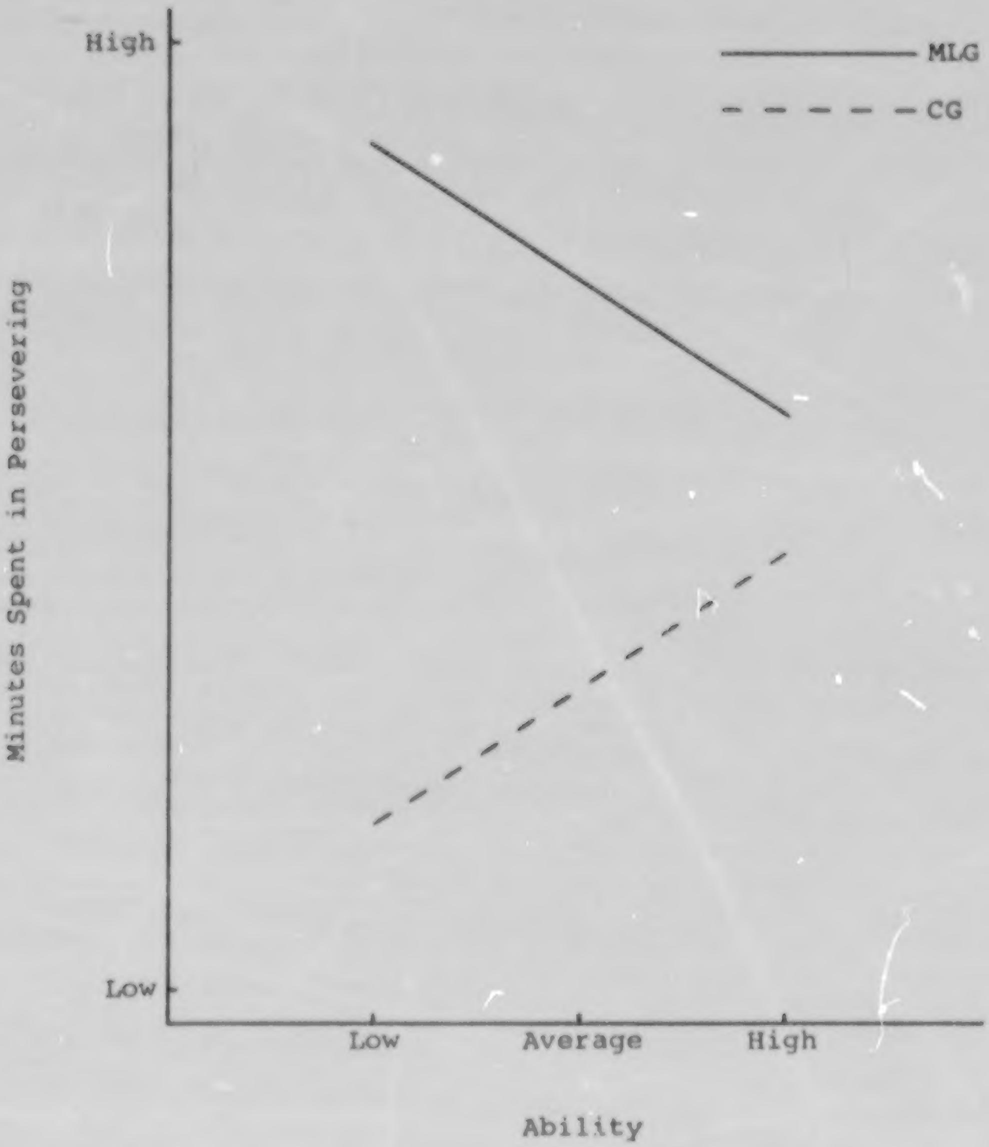

Fig. 4. Illustration of the hypothesized ordinal interaction between ability to understand instruction and quality of instruction relative to the number of minutes spent in persevering.

Experimental consequence. The relationship between the number of minutes spent in persevering on a difficult learning task and ability to understand instruction will be of a significantly inverse nature in the mastery learning group but of a significantly direct nature in the control group, and these relationships will differ significantly from each other.

Where $r_{PAb,MLG}$ = the correlation between number of minutes spent in persevering on a difficult learning task and ability for the mastery learning group

and

$r_{PAb,CG}$ = the correlation between number of minutes spent in persevering on a difficult learning task and ability for the control group,

it is predicted that:

1. $r_{PAb,MLG} < 0$
2. $r_{PAb,CG} > 0$
3. $r_{PAb,CG} - r_{PAb,MLG} \neq 0$

Research hypothesis 14. With adjustments made for differences in degree of learning, there will be a significant difference between the mastery learning group and the control group relative to perseverance. More specifically, with adjustments made for differences in degree of learning, the mastery learning group will manifest a significantly greater amount of perseverance than will the control group.

Experimental consequence. With adjustments made for differences in achievement scores on a summative posttest, the mastery learning group will spend a significantly greater number of minutes in persevering on a difficult learning task than will the control group.

Where $MP(adj.)_{MLG}$ = the mean number of minutes spent by the mastery learning group in persevering on a difficult learning task with adjustments made for differences in achievement scores on the summative posttest

and

$MP(adj.)_{CG}$ = the mean number of minutes spent by the control group in persevering on a difficult learning task with adjustments made for differences in achievement scores on the summative posttest,

it is predicted that:

$$MP(adj.)_{MLG} > MP(adj.)_{CG}$$

Research hypothesis 15. With adjustments made for differences in degree of learning, there will be a significant ordinal interaction between ability to understand instruction and quality of instruction relative to perseverance. More specifically, with adjustments made for differences in degree of learning, as students decrease in ability to understand instruction, the amount of perseverance manifested will increase in the mastery learning group but will decrease in the control group.

Experimental consequence. With adjustments made for differences in achievement scores on a summative posttest, the control group's number of minutes spent in persevering on a difficult learning task will be more severely limited for students low in ability to understand instruction than for students high in ability to understand instruction. (See Figure 5 for an illustration of the hypothesized ordinal interaction between ability to understand instruction and quality of instruction relative to the number of minutes spent in persevering with adjustments made for differences in summative posttest achievement scores.)

Where $MP(adj.)_{MLG,LAb}$, $MP(adj.)_{MLG,AAb}$, and $MP(adj.)_{MLG,HAb}$ = the mean number of minutes spent in persevering on a difficult learning task by the mastery learning group which is divided into low-, average-, and high-ability levels, respectively, with adjustments made for differences in achievement scores on the summative posttest

and

$MP(adj.)_{CG,LAb}$, $MP(adj.)_{CG,AAb}$, and $MP(adj.)_{CG,HAb}$ = the mean number of minutes spent in persevering on a difficult learning task by the control group which is divided into low-, average-, and high-ability levels, respectively, with adjustments made for differences in achievement scores on the summative posttest,

it is predicted that:

$$MP(adj.)_{MLG,LAb} - MP(adj.)_{CG,LAb} > MP(adj.)_{MLG,AAb} - MP(adj.)_{CG,AAb} > MP(adj.)_{MLG,HAb} - MP(adj.)_{CG,HAb}$$

Research hypothesis 16. With the effects due to differences in degree of learning partialed out, there will be a significant difference between the mastery learning group and the control group relative to the correlation between perseverance and ability to understand instruction. More specifically, with the effects due to degree of learning partialed out, the correlation between perseverance and ability to understand instruction will be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations will differ significantly from each other.

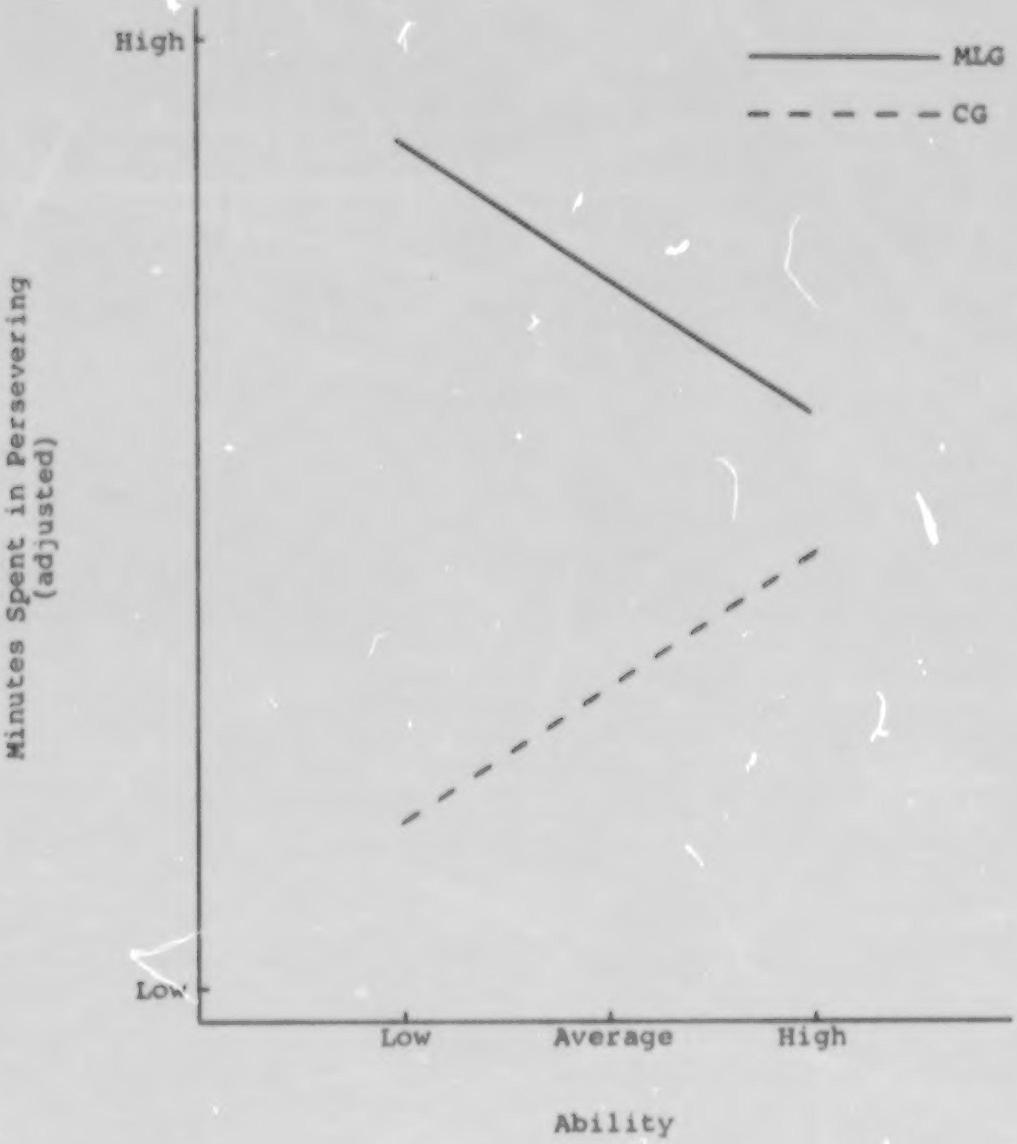

Fig. 5. Illustration of the hypothesized ordinal interaction between ability to understand instruction and quality of instruction relative to the number of minutes spent in persevering with adjustments made for differences in summative posttest achievement scores.

Experimental consequence. With the effects due to achievement scores on a summative posttest partialed out, the relationship between the number of minutes spent in persevering on a difficult learning task and ability to understand instruction will be of a significantly inverse nature in the mastery learning group but of a significantly direct nature in the control group, and these relationships will differ significantly from each other.

Where $r_{PAb.Ac,MLG}$ = the partial correlation between number of minutes spent in persevering on a difficult learning task and ability with adjustments made for differences in achievement scores on the summative posttest for the mastery learning group

and

$r_{PAb.Ac,CG}$ = the partial correlation between number of minutes spent in persevering on a difficult learning task and ability with adjustments made for differences in achievement scores on the summative posttest for the control group,

it is predicted that:

1. $r_{PAb.Ac,MLG} < 0$
2. $r_{PAb.Ac,CG} > 0$
3. $r_{PAb.Ac,CG} - r_{PAb.Ac,MLG} \neq 0$

Research hypothesis 17. There will be a significant difference between the mastery learning group and the control group relative to the correlation between degree of learning and perseverance. More specifically, the correlation between degree of learning and perseverance will be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations will differ significantly from each other.

Experimental consequence. The relationship between achievement scores on a summative posttest and the number of minutes spent in persevering on a difficult learning task will be of a significantly inverse nature in the mastery learning group but of a significantly direct nature in the control group, and these relationships will differ significantly from each other.

Where $r_{AcP,MLG}$ = the correlation between achievement scores on a summative posttest and number of minutes spent in persevering on a difficult learning task for the mastery learning group

and

$r_{AcP,CG}$ = the correlation between achievement scores on a summative posttest and number of minutes spent in persevering on a difficult learning task for the control group,

it is predicted that:

1. $r_{AcP,MLG} < 0$
2. $r_{AcP,CG} > 0$
3. $r_{AcP,CG} - r_{AcP,MLG} \neq 0$

CHAPTER 2

Review of the Literature

Mastery Learning and the Carroll Model As Viewed from an Historical Perspective

A consideration of the historical antecedents of any educational practice can undoubtedly serve to enhance the awareness and understanding which one brings to bear upon its genesis, development, and implications for current practice. The attention presently being directed toward such topics as mastery learning and the Carroll model by theorists and practitioners alike is certainly no exception.

As early as the 1920's evidence can be found of attempts by educators to formulate curricular programs and instructional strategies in such a way as to enhance the probability of content mastery being attained by the vast majority of students. The contributions of Carleton Washburne (1922) in the Winnetka Plan and Henry C. Morrison (1926) at the University of Chicago Laboratory School are two classic examples of such endeavors.

As Superintendent of Schools in Winnetka, Illinois, Washburne (1922) developed an educational program that represented a break from the conventional practice of allowing time units to serve as the constant in a school setting while student achievement fluctuated according to individual ability. More specifically, he sought to make units of achievement the constant while permitting the time factor to vary in accordance with the individual capabilities of the students. In order to effect this shift in emphasis from time units to achievement units, the following steps were deemed necessary by Washburne (pp. 198, 200, 206):

1. Establishment of goals or subject-matter units of achievement which must be mastered in sequential order

2. Construction of unit achievement tests which would diagnose the weakness of each student relative to lack of knowledge

3. Preparation of self-corrective practice exercises which would help students remediate those deficiencies identified by the diagnostic tests.

Student learning under the Winnetka Plan, then, was of a self-paced nature; that is, each student was allotted whatever amount of time he needed in order to attain mastery.

Morrison (1926), likewise, was sensitive to the emphasis assigned to the time and achievement factors in a school situation. From his perspective, "a given series of essential learnings is not necessarily acquired in a given restricted time merely because such would be administratively convenient. The constant is the learning; the variable is the time required (p. 69)." Consistent with this view of learning as the constant and time as the variable, Morrison suggested the following pedagogical technique which he identified as a mastery formula: "Pre-test, teach, test the result, adapt procedure, teach and test again to the point of actual learning (p. 81)." It was intended that feedback from the testing aspect of the mastery formula would permit the teacher to make one of the following determinations: (a) the teaching act was successful; hence, the teacher could then proceed to the next learning unit, or (b) the teaching act was unsuccessful; hence, a modification of the instructional procedure was needed prior to reteaching.

According to Morrison's mastery approach, then, each student was allotted whatever amount of time was required by his teacher to bring the majority of students to mastery.

In a summarization of the major features permeating both the Winnetka Plan and Morrison's approach, Block (1971) identified the following six commonalities:

1. Educational objectives were used to specify exactly what the student was expected to master.

2. Specific learning units formed the basis around which instruction was organized.

3. Each learning unit had to be mastered in a sequential fashion.

4. Feedback from ungraded, diagnostic tests revealed the adequacy of the student's learning at the completion of each unit.

5. Contingent upon the diagnostic feedback, appropriate learning correctives were specified as a supplement to the original instruction.

6. The time factor was considered the variable in the attempt to individualize instruction and thereby foster mastery learning.

In his discussion of the history of mastery learning, Block (1971) made the following points: Mastery strategies persisted well into the 1930's but eventually disappeared due to the failure of educational technology to support such a methodological approach. The idea of mastery learning finally reappeared, however, during the late 1950's in association with programed instruction. Although programed instruction did function as a valuable tool for some students by virtue of its frequent drill and reinforcement, it did not provide a useful model upon which could be based a mastery learning strategy for the vast majority of students.

The publication of John B. Carroll's (1963) model of school learning, however, provided a conceptual paradigm which later was to serve as a catalyst for the resurgence of mastery learning strategies. Reflecting upon the genesis of his model, Carroll (1970) noted that it was originally developed in the context of his work on the prediction of success in foreign language training. More specifically, Carroll had discovered that it was ". . . possible to state approximately how much training time would be needed by a person with a given level of foreign language aptitude to get to a given level of proficiency in a given foreign language (p. 40)." Carroll (1962) thought it desirable to develop a model which would encompass not only the variables of aptitude and motivation but also variables related to the instructional process and which would not be restricted to the area of foreign language learning. He identified the appropriate variables as follows:

> *Instructional Variables*.
>
> p_j = *adequacy of presentation of task j* (on a scale from 0 to 1). . . .
>
> o_j = *the time allowed, "opportunity," for learning task j*. . . .
>
> *Individual Difference Variables*.
>
> g_i = *that characteristic, general intelligence or verbal intelligence, which determines the extent to which the individual will be able to understand directions and explanations or to infer such directions and explanations from the total content of the instruction even when they are lacking*. . . .
>
> a_{ij} = *the time which would be needed by individual i to learn task j to a specified criterion of learning*, on the assumption that . . . the task is presented well enough for him to understand the task in the light of his g_i). . . .
>
> m_{ij} = *the maximum amount of time individual i would apply himself to the learning of task j* (pp. 121-122).

A brief recall of the Carroll model as presented in Chapter 1 will make it readily apparent that the two instructional variables correspond to the components of *quality of instruction* and *opportunity* while the three individual difference variables correspond to the components of *ability to understand instruction*, *aptitude*, and *perseverance*, respectively.

As alluded to earlier, it was Carroll's model which served as a catalyst for the resurgence of interest in and subsequent development of a mastery learning strategy by B. S. Bloom and his associates at the University of Chicago. Bloom (1968) acknowledged that there are numerous alternative strategies for mastery learning (for example, tutoring, self-pacing, tracking or streaming, and nongrading); however, it is his contention that each must incorporate some way of accommodating the five variables contained in the Carroll model.

The particular mastery learning strategy developed recently at the University of Chicago by Bloom and his colleagues can be described in the following manner: The preconditions of the strategy encompass the specification of objectives and subject-matter content as well as the translation of these into summative evaluation procedures with absolute performance standards established (Bloom, 1968). The operating procedures of the strategy center around (a) the use of ungraded, diagnostic-progress tests which are designed to provide formative feedback to students and teachers and (b) the specification of learning correctives which are based upon the diagnostic feedback and, therefore, function to remedy specific learning deficiencies (Block, 1971).

In summation, then, the Bloom strategy for mastery learning is one which serves ". . . to supplement regular group instruction by using diagnostic procedures and alternative instructional methods and materials in such a way as to bring a large proportion of the students to a predetermined standard of achievement (Bloom, 1968, p. 7)." Despite the great similarity between this procedure and the earlier mastery learning approaches of Washburne (1922) and Morrison (1926), Block (1971) contended that the Bloom strategy for mastery learning is a great advance over previous strategies by virtue of its access to improved formative feedback instruments and a greater variety of learning correctives.

Current Status of Research Related to the Operations of the Carroll Model

Feedback/Correction Procedures As a High Quality of Instruction

As was noted in the previous section, Carroll's model of school learning has served as a catalyst and framework for the development of feedback/correction strategies for mastery learning. Furthermore, research studies which employed the aforementioned strategy but which were conducted prior to either Carroll's publication of the conceptual model (1963) or Bloom's transformation of it into a working paradigm (1968) have also been interpreted in terms of the model. Irrespective of the timing of the research, though, conclusive findings have been reported which tend to justify feedback/correction procedures as a high quality of instruction capable of increasing substantially the degree of learning on the part of students (Airasian, 1967; Baley, 1972; Block, 1970; Carroll & Spearritt, 1967; Collins, 1969, 1970; Gentile, 1970; Keller, 1968; Kersh, 1970; Kim et al., 1969, 1970; Mayo, Hunt, & Tremmel, 1968; Merrill, Barton, & Wood, 1970; Moore, Mahan, & Ritts, 1968; Sherman, 1967; Silberman & Coulson, 1964; Thompson, 1941) as well as the efficiency of their time spent in learning (Airasian, 1967; Block, 1970; Carroll & Spearritt, 1967; Merrill et al., 1970).

Investigations of the Interaction Effects of Ability To Understand Instruction and Quality of Instruction Upon Degree of Learning and/or Time Spent

The practice of basing the development of an instructional strategy upon a conceptual paradigm such as Carroll's model is certainly a laudable attempt by educators to explain and justify the rationale behind their actions. Far too often, though, the theoretical assumptions contained either implicitly or explicitly in Carroll's model have been accepted uncritically as valid and, thus, have not been subjected to rigorous empirical verification (Gaines, 1971, 1973). Carroll himself has insisted that the various components and operations of his model are in need of further investigation (1963, 1970). Indeed, his own research has even suggested the direction of possible modifications of the model (Carroll & Spearritt, 1967).

One of the most interesting aspects contained in the Carroll model concerns the hypothesized interaction effects of ability to understand instruction and quality of instruction upon degree of learning and/or perseverance. The nature of this hypothesized interaction, as explicitly stated relative to degree of learning and logically inferred relative to perseverance, is such that students low in ability to understand instruction will suffer more when subjected to low quality of instruction than will students high in ability to understand instruction.

Given the definitions of ability to understand instruction and quality of instruction as specified by Carroll (see

Chapter 1, pp. 7-8), the aforementioned interaction avails itself of that genre of investigation known in the literature as aptitude-treatment interaction (ATI) research. In relating Carroll's model to the aptitude-treatment interaction concept, Gaines & Jongsma (1973) made the following observations: Quality of instruction is synonymous with what is commonly referred to as treatment. There are two dimensions along which treatment may profitably be defined. The first dimension includes those factors that would be expected to foster a high quality of instruction for all learners; the second dimension, however, encompasses those factors that would be expected to promote a high quality of instruction for learners on an individual basis. With respect to the variable labeled ability to understand instruction, this expression is synonymous with terms such as complex or generalized aptitude. The use of the expression aptitude in the context of the model, though, is intended to connote the same meaning frequently associated with the terms simple or task-specific aptitude. It follows, then, that

> Carroll's notion of aptitude . . . is conceptually related to the dimension of quality of instruction that accommodates the individual learner's special needs and characteristics. On the other hand, Carroll's notion of ability to understand instruction is conceptually related to the dimension of quality of instruction that involves factors which are generalizable across learners (p. 6).

Despite the obvious necessity for basic research on the validity of Carroll's model as well as the amenability of the model to a research approach of the aptitude-treatment interaction type, the literature is currently limited to only two studies which were designed intentionally to investigate Carroll's hypothesized interaction (Carroll & Spearritt, 1967; Gaines, 1971) and two studies which lend themselves to interpretation in terms of the hypothesized interaction (Kim et al., 1969; Silberman & Coulson, 1964). In each of these four studies some manifestation of a feedback/correction procedure was employed on the assumption that it represented a high quality of instruction for all learners. Consistent with this view of a treatment generalizable across learners was the identification of some type of complex or generalized aptitude for each student. The remaining paragraphs of this section endeavor to elaborate upon the specifics of each of these four investigations.

Carroll & Spearritt (1967) attempted to assess the interactive effects of ability to understand instruction and quality of instruction upon two dependent variables: time to criterion on the main learning task and perseverance on a difficult post-experimental task. The sample consisted of 208 sixth graders who were categorized into three levels of IQ: high, above average, and average-low. For the purpose of teaching an artificial foreign language known as "Midimo," two forms of a self-instructional booklet were prepared and randomly assigned to the students within each level of ability. Form A represented a high quality of instruction by virtue of its inclusion of a highly organized and sequential presentation of content, formative testing for the purpose of identifying learning errors, and the explanation of the errors via correction/review procedures. Form B represented a low quality of instruction in that it involved the disorganized presentation of all content simultaneously and the absence of correction/review procedures that could have served to explain those errors identified by the formative testing.

Among the research findings and conclusions resulting from this study were the following:

1. There was no significant interaction between IQ and quality of instruction relative to the time taken to reach criterion on the main learning task. Even when adjustments were made for differences in perseverance on the difficult post-experimental task, the same finding of no significant interaction resulted. Contrary to the prediction of the model, then, high ability students were just as much affected by poor quality of instruction as were students of average and low ability.

2. There was a significant disordinal interaction ($p < .05$) between IQ and quality of instruction relative to perseverance on a difficult post-experimental task. The direction of this interaction was such that those students exposed to the low quality of instruction were less willing to spend time on a difficult post-experimental task if they were in the high- or low- but not middle-ability level. This finding is not consistent with the hypothesized ordinal interaction of ability to understand instruction and quality of instruction relative to degree of learning or time needed.

Hence, it could be inferred that a modification of the model might be needed, even if only in reference to the manner in which ability to understand instruction and quality of instruction interact to influence perseverance, or time spent.

In recognition of the dire need for research on the operations of Carroll's model, Gaines (1971) investigated the interaction between ability to understand instruction and quality of instruction relative to degree of learning. Based upon a sample of 28 classes ranging from the fifth through the eighth grades, six levels of reading achievement were identified. Two variations of a mastery learning strategy were developed for and implemented in the teaching of a social studies unit on anthropology. The first variation, Treatment 1, was regarded as the high quality of instruction. This treatment encompassed such items as performance objectives, specific feedback from mastery tests of a formative nature, and correction procedures via prescriptive rereading and/or reteaching. The second variation, Treatment 2, was characterized as low quality of instruction. This approach involved the use of performance objectives and general feedback from workbooks which provided self-correcting exercises.

The findings and conclusions of this study which relate directly to the hypothesized interaction were as follows: There was no significant interaction between reading level and treatment relative to degree of learning. Given the context and limitations of the study, especially the lack of sufficiently sharp distinctions between treatments, it was not possible to confirm Carroll's hypothesized interaction.

While not designed with the intention of investigating the operations of Carroll's model, a study conducted by Kim et al. (1969) provided findings which can be interpreted in terms of the interactive effects of ability to understand instruction and quality of instruction upon degree of learning. Working with a sample of 272 seventh-graders who were classified into three levels of intelligence, an attempt was made to contrast the effectiveness of mastery and non-mastery learning strategies for the teaching of a unit on geometry. The mastery or experimental group could be considered the recipient of a high quality of instruction in that it was exposed to an instructional strategy consisting of the following elements: performance objectives; feedback from formative tests with a criterion level of 80 percent; and learning correctives by way of remedial programed instruction, review questions, and student tutors. The non-mastery or control group, however, received only the performance objectives; hence, this group could be viewed as having received a low quality of instruction.

Included among the findings reported by Kim et al. were the following: For those students with below-average intelligence, 50 percent of the mastery group as compared to only 8 percent of the non-mastery group attained the criterion level set for a summative achievement test. With respect to those students whose intelligence quotient placed them at the above-average level, 95 percent reached the criterion level while working under mastery conditions as compared to only 64 percent of the non-mastery group.

Although the findings cited immediately above were not subjected to a statistical test of interaction, they did suggest that students low in ability to understand instruction suffered more when subjected to a low quality of instruction than did students high in ability to understand instruction. This pattern would have provided evidence in support of Carroll's hypothesized interaction had it been subjected to statistical analysis and found to be significant.

Silberman & Coulson (1964), likewise, reported a study which did not investigate Carroll's model but which did provide evidence potentially supportive of Carroll's hypothesized interaction. This report described the use of a tutorial strategy for evaluating and revising instructional programs in high school geometry, junior high school Spanish, and first-grade reading and arithmetic. Whenever a student manifested deficiencies in a given program, tutorial assistance was provided. This type of intervention contributed to a collection of records indicating common student difficulties encountered as well as the specific tutorial correctives that remedied them. Those correctives which proved to be effective were included in periodic revisions of each program. After several such revisions a comparison was made of the final version and the original version of each program.

Hence, each final version could be thought of as a high quality of instruction in that it contained components which had been demonstrated to be effective in correcting common sources of confusion for students. In contrast, each original version could be viewed as a low quality of instruction in that it was void of any component that was the result of formative evaluation procedures.

The findings which emanated from the comparison of the final version and the original version of each program demonstrated that the mean student achievement was significantly higher for students working with the revised program. Furthermore, it was observed that the more intelligent students tended to be less affected by deficient programs than were the less intelligent students. Although this latter finding is reminiscent of the interaction hypothesized by Carroll, the data were not subjected to a statistical test of interaction and, hence, cannot be judged as supportive or nonsupportive of that operation of the model.

The research findings contained in the four studies just reviewed did not represent conclusive evidence which could be used to support or reject completely Carroll's hypothesized interaction between ability to understand instruction and quality of instruction. More specifically, the various investigations which were summarized served to fulfill the following purposes: to indicate the absence of a significant interaction of the type hypothesized by Carroll (Carroll & Spearritt, 1967; Gaines, 1971); to identify the presence of a significant interaction suggestive of possible modifications needed in the model (Carroll & Spearritt, 1967);

to present data consistent with the pattern of interaction hypothesized by Carroll but lacking in a statistical test of significance (Kim et al., 1969; Silberman & Coulson, 1964).

Investigations of the Correlational Aspects Inherent in the Carroll Model

Research studies which seek to demonstrate the efficacy of feedback/correction procedures, as well as those which endeavor to test Carroll's hypothesized interaction, do not exhaust the scope of investigations that can be performed in relation to the Carroll model. Another path of investigation which more often than not arises as a corollary of a much larger study is that which considers the direction and size of correlations between the various components of Carroll's model.

Gaines (1971) and Baley (1972) reported findings in their studies which correlated degree of learning with ability to understand instruction. Both of these investigators relied upon feedback/correction procedures as the basis for establishing a dichotomy between high and low qualities of instruction and then proceeded to identify the correlation coefficients that resulted in each instance.

More specifically, Gaines (1971) reported the following correlation coefficients between anthropology posttest scores and reading achievement scores across two treatment levels and four grade levels (fifth, sixth, seventh, and eighth): high quality of instruction, .30, .64, .78, and .64; low quality of instruction, .34, .71, .77, and .70. Seven of the eight coefficients were significant at the .05 or .01 level; however, there was no significant difference between the two treatment groups relative to the correlation investigated. The suggestion, then, was that ". . . the relationship between reading achievement scores and posttest scores may not have been differentially affected by treatments (p. 93)."

In the case of Baley (1972), the following correlation coefficients between quantitative ability scores and algebra achievement scores were cited for each of two treatment levels: high quality of instruction, .25; low quality of instruction, .57. No statistical tests of significance were reported. The aforementioned data supported the researcher's hypothesis that a high quality of instruction would result in a correlation coefficient which approached zero and which would be less than the coefficient recorded under a low quality of instruction.

Airasian (1967) conducted an investigation which provided data relative to the correlation between total hours spent in weekly study and achievement scores attained in a graduate level course on test theory. Reporting only on the findings emanating from a mastery learning group which had the benefit of feedback/correction procedures, coefficients for each of the ten weeks of the study were cited as ranging from .07 to -.46.

Lewis (1969), while not defining his high quality of instruction in terms of feedback/correction procedures, also tested the hypothesis that the correlation between general intelligence and degree of learning would be greater at a low level of quality of instruction than at a higher level. His findings, however, gave evidence of the exact opposite. The correlation coefficients that resulted under the high quality of instruction and the low quality of instruction were .80 and .55, respectively. Furthermore, the difference between the two treatment groups relative to the coefficients just cited was significant at the .10 level.

CHAPTER 3

Methodology and Procedures

Description of the Sample

The sample used in this investigation was identical to a population of 169 male students who were enrolled in the second of six learning sequences (units) which comprised an Algebra I course at the secondary school level. The nongraded pattern of school organization under which these students functioned resulted in the sample not being restricted to only those individuals from a given class level (for example, freshman, sophomore, junior, or senior).

Intelligence quotient scores for this group as attained on the _California Short-Form Test of Mental Maturity_, 1963 Revision, ranged from 81 to 133. Based upon descriptive categories for classifying scores, as recommended by the _Guide to Interpretation_ which accompanied the aforementioned instrument, three levels of ability were identified and labeled as follows: high (108 and above), average (93 to 107), and low (92 and below). The high-, average-, and low-ability levels contained 68, 72, and 30 students, respectively.

Experimental mortality which occurred during the implementation of this study (from September 24, 1973 to December 7, 1973), however, resulted in the reduction of the sample size from 169 to 141 students. The loss of 28 students from the initial sample of 169 was attributed to two causes: (_a_) the correction of scheduling assignments which erroneously had placed some students in the second learning sequence in algebra (entitled Learning Sequence 332) and (b) the withdrawal of some students from Learning Sequence 332 for the purpose of rescheduling a learning sequence in general mathematics or business mathematics.

Hence, the statistical analyses performed on the data collected in this study were based upon a final sample size of 141 students. The number of students distributed across the high-, average-, and low-ability levels were 60, 59, and 22, respectively.

Treatments

The students within each of the three ability levels cited above were randomly assigned to two levels of treatment: the mastery learning treatment and the control treatment. Initially, there were 85 and 84 students in the mastery learning group and the control group, respectively. As a result of the experimental mortality described above, the size of the mastery learning group and the control group was reduced to 64 and 77 students, respectively. As alluded to earlier, the students in this study were exposed to algebraic topics contained within a learning sequence or unit entitled as follows: "Learning Sequence 332: Polynomials and Equations." More specifically, the first part of this learning sequence, which concerned the addition, subtraction, multiplication, and division of directed numbers, represented the material upon which the two treatments were based.

Due to the nongraded, continuous-progress approach to instruction which comprised the educational setting for the students in this study, "instructional packets" were employed as the primary means to organizing and presenting new material. Appendices A and B contain the instructional packets and, hence, the vehicles for the instructional treatments to which the mastery learning group and the control group, respectively, were exposed. The following three sections endeavor to elaborate upon the commonalities as well as the distinguishing components of each treatment.

Commonalities Permeating Both the Mastery Learning Treatment and the Control Treatment

The following components were common to the instructional treatments under which both the mastery learning group and the control group functioned:

1. A description of content which informed the student of those specific mathematical topics to which he would be exposed

2. A statement of performance objectives which apprised the student of those particular learning tasks which he was responsible for mastering

3. A specification of assignments which corresponded to each performance objective and which represented learning activities required of each student

4. An announcement of the point at which a summative posttest must be taken for the purpose of demonstrating the level of achievement attained

5. A collection of instructional materials which supplemented the explanations given in the student's text and to which the student was referred in his assignments

6. The provision of answer keys not contained within the student's text.

Distinguishing Characteristics of the Mastery Learning Treatment

The two elements of the mastery learning treatment which were responsible for its being representative of a high quality of instruction included the following: (*a*) the use of formative trial tests and (*b*) the prescription of learning correctives of a review-remedial nature.

Formative trial tests. Students in the mastery learning group were subjected to formative trial tests at two intermittent stages prior to the completion of the first part of Learning Sequence 332. More specifically, Formative Trial Test I was administered immediately subsequent to the completion of the assignments associated with the addition and subtraction of directed numbers (performance objectives 1 through 4) while Formative Trial Test II was administered immediately after the completion of the assignments associated with the multiplication and division of directed numbers (performance objectives 5 through 8).

The rationale behind the use of this type of formative evaluation was that the trial tests would provide both the student and the teacher with on-going feedback relative to the learning deficiencies experienced by the student as well as the alterations most needed in the instructional materials and/or strategies.

For the purpose of ensuring the effectiveness of each formative trial test, a criterion level of 80 percent mastery was established. A score of 80 or higher on a 100-point scale entitled the student to proceed to the next group of objectives (in the case of Formative Trial Test I) or to the summative posttest (in the case of Formative Trial Test II). A score which indicated less than 80 percent mastery, however, required that the student retake the trial test (that is, a different form of it) until 80 percent mastery was demonstrated. In either situation, though, any test items missed served to determine those learning correctives to which the student was recycled.

Appendices C and E encompass the six forms of Formative Trial Test I and Formative Trial Test II, respectively, which were administered to members of the mastery learning group.

Learning correctives. Based upon the feedback provided by the formative trial tests, exercises of a review-remedial nature were prescribed for the purpose of correcting any learning deficiencies experienced by a student. Upon the initial unsuccessful attempt at attaining the criterion score on either trial test, the student was recycled back to additional assignments of the same type as those he had completed previously. However, upon any unsuccessful attempts thereafter at attaining the criterion score on either trial test, the student was provided with a brief tutoring session in which a teacher or paraprofessional attempted to provide the needed personalized assistance.

Appendices D and F include the learning correctives which were based upon the results of Formative Trial Test I and Formative Trial Test II, respectively, and which were employed in the mastery learning treatment.

Distinguishing Characteristics of the Control Treatment

The characteristic which was most instrumental in the control treatment being defined as a low quality of instruction was the complete absence of formative trial tests and the corresponding learning correctives. Although the members of the control group did have the benefit of the five commonalities identified earlier, the most crucial, perhaps, being the specification of performance objectives, they were required to complete all assignments and to take the summative posttest without the assistance of the formal feedback/correction procedures described above.

Concluding Remarks Relative to the Setting in Which the Treatments Were Implemented

At this point it is imperative that acknowledgment be given to the following facts which pertain to the setting in which the two aforementioned treatments were implemented:

1. The sample of 141 students who were used in this study spanned a total of three of the four class periods during which the Algebra I course was offered. Contained within each of these three classes were members of _both_ the mastery learning group and the control group. The only

distinctions made between the members of the two groups were in terms of (a) the contents of the instructional packets and (b) the process of formative trial testing and the completing of learning correctives.

2. The mastery learning treatment and the control treatment were implemented in a nongraded setting which incorporated independent study, small-group instruction, and large-group motivational lectures. Of these three approaches for organizing instruction, though, the reliance upon independent study far outweighed the attention given to small-group instruction and large-group motivational lectures. The independent study was conducted in a large open area with teachers and paraprofessionals circulating among the students for the purpose of aiding those who were in need of assistance. Quite obviously, then, members of the control group, as well as those of the mastery learning group, had easy access to this informal type of feedback/correction procedure.

3. The major components of the control treatment represent the standard operating procedures which already had been planned for implementation even before this researcher's arrival at the site of the study.

4. The major components of the mastery learning treatment, however, represent this researcher's alteration of the school's standard operating procedure for the purpose of constructing an instructional strategy more conducive to the attainment of mastery learning by a greater percentage of students.

Development of Instruments

As a result of the nongraded setting in which the study was conducted, students in the sample began and completed the first part of Learning Sequence 332 in an extremely staggered fashion. In anticipation of this occurrence, multiple forms of the formative trial tests, as well as the summative post-test, were developed. The sections which follow endeavor to explain (a) the procedures taken in the preparation of these two types of instruments and (b) their criterion-referenced orientation. Also, the instrument labeled "Assessment of Perseverance" is introduced and its development traced.

Formative Trial Tests I and II

Test construction. Five forms of Formative Trial Test I (see Appendix C), as well as five forms of Formative Trial Test II (see Appendix E), were developed. As mentioned in a preceding section, it was intended that Formative Trial Tests I and II be administered immediately subsequent to the completion of the assignments associated with (a) the addition and subtraction of directed numbers and (b) the multiplication and division of directed numbers, respectively. Hence, performance objectives 1 through 4 served as the basis for the construction of Formative Trial Test I while performance objectives 5 through 8 functioned as the basis for the development of Formative Trial Test II.

The actual development of the five forms of each formative trial test proceeded in accordance with four specific steps. These procedures, as applied to the construction of Formative Trial Test I, are discussed below:

1. It was recognized that the essence of performance objectives 1 and 2 called for a knowledge of and the ability to apply the rule for finding the sum of directed numbers involving any combination of algebraic signs. Similarly, it was recognized that the essence of performance objectives 3 and 4 focused upon a knowledge of and the ability to apply the rule for subtraction of directed numbers involving any combination of algebraic signs.

2. A decision was made to include a total of ten items on Formative Trial Test I. Items 1 through 5 would be based upon performance objectives 1 and 2 while items 6 through 10 would be based upon performance objectives 3 and 4.

3. Performance objectives 1 and 2 referred to the subject matter contained in sections 4-3 and 4-6 of the algebra test (Dolciani, Berman, & Freilich, 1962) used by the students. Performance objectives 3 and 4 pertained to the algebraic topic encompassed within sections 4-4 and 4-7 of the same text. Founded upon a close scrutiny of the type and complexity of problems treated in these sections of the text, test items 1 through 5 and 6 through 10 were constructed in accordance with the specifications of performance objectives 1 and 2 and performance objectives 3 and 4, respectively. Obviously, no test item was written that was a duplicate of a problem used as an example or given as an assignment in the text itself.

4. The three steps just cited represent the procedures which were taken in the construction of Form A of Formative Trial Test I. In addition to this first form, Forms B through E of Formative Trial Test I were also developed. The construction of the test items on these last five forms resulted from the simple process of varying the numbers which comprised each item on Form A. Careful attention, however, was devoted to ensuring that (a) no given test item appeared more than once among any of the forms and (b) the algebraic signs associated with the numbers in the initial statement of the problem, the steps needed for its solution, and the final answer did not differ whatsoever for corresponding items across the five forms.

As alluded to earlier, the four procedural steps that were implemented in the development of the five forms of Formative Trial Test I were also employed in the construction of the five forms of Formative Trial Test II. In this latter case of Formative Trial Test II, though, two differences of a rather obvious nature were in effect: (a) the essence of performance objectives 5 and 6 called for a knowledge of and ability to apply the rule for multiplying directed numbers involving any combination of algebraic signs while the essence of performance objectives 7 and 8 focused upon a knowledge of and ability to apply the rule for dividing directed numbers involving any combination of algebraic signs and (b) the sections (4-4 and 4-7) of the algebra text to which performance objectives 5 through 8 pertained, and upon which the type and difficulty of the test items were based, were those which concerned the multiplication and division of directed numbers.

Content validity. There existed a direct correspondence between the various test items and the performance objectives upon which they were based relative to subject-matter content, required student behavior, and the conditions under which the behavior had to be demonstrated. Hence, the assumption was made by this researcher that the content validity of the various forms of the two formative trial tests was ensured. This assumption was corroborated by the professional judgment of the three certified mathematics teachers whose students were involved in the study.

Summative Posttest

Test construction. Two forms of the summative posttest (see Appendix G) were developed. As was the case with the construction of the formative trial tests, the development of the summative posttest was based upon those performance objectives which spanned the entire range of topics for which the students were to be held accountable. More specifically, the test items which comprised the two forms of the summative posttest were founded upon performance objectives 1 through 8.

The actual development of Summative Posttest Form I proceeded in accordance with the same procedures utilized in the construction of the formative trial tests. For purposes of review and adaptation to the special case of the summative posttest, these steps can be summarized as follows:

1. Acknowledgment was given to the particular learning task or tasks which comprised the essence of each performance objective. In the case of Summative Posttest Form I, this acknowledgment was directed toward performance objectives 1 through 8.

2. A decision was made relative to the number of test items that should be included on the instrument. With respect to Summative Posttest Form I, 25 items were included for the following reasons: (a) Given the time limitations of a 55-minute class period as well as the algebraic topics being considered, a total of 25 test items was viewed as the maximum number that could be administered and still facilitate the "power" dimension instead of the "speed" dimension of the instrument. (b) Given the division of the first part of Learning Sequence 332 into the four topical areas of adding, subtracting, multiplying and dividing directed numbers as well as the specification of two performance objectives per topical area, the allotment of approximately six test items per topical area easily served to accommodate the limit of 25 items previously discussed. The actual number of test items included on Summative Posttest Form I and the performance objectives which they represented were as follows: Items 1 through 8, performance objectives 1 and 2; items 9 through 13, performance objectives 3 and 4; items 14 through 19, performance objectives 5 and 6; and items 20 through 25, performance objectives 7 and 8.

3. The actual construction of each test item was performed not only in accordance with the content, behavioral,

and conditional specifications of the pertinent performance objective but also subsequent to a close examination of the type and difficulty of problems treated in those sections of the student's text which related to the topic being represented. Obviously, no test item was written that was a duplicate of a problem used as an example or given as an assignment in the text itself.

4. In addition to Summative Posttest Form I, a second form of the instrument was also developed. The construction of the test items on Summative Posttest Form II resulted from the simple process of varying the numbers which comprised each item on the first form. Careful attention, however, was devoted to ensuring that (a) no given test item appeared on both forms of the summative posttest and (b) the algebraic signs associated with the numbers in the initial statement of the problem, the steps needed for its solution, and the final answer did not differ whatsoever for corresponding items across the two forms. Furthermore, attention was given to guaranteeing that no test item appeared on either form of the summative posttest which had already been included on one of the forms of Formative Trial Tests I or II.

Although two forms of the summative posttest were constructed, it was deemed necessary by the researcher to project the image to the students in the study that a greater number of forms were being used. The rationale for this decision rested upon the assumption that due to the nongraded nature of the school the 141 students in the sample would begin and complete the first part of Learning Sequence 332 in a staggered fashion. This assumption later proved to be very much the case. The implications of this factor relative to the security of the test items which comprised Summative Posttest Form I and Summative Posttest Form II are obvious.

The decision was made, then, to derive two additional "forms" (Form III and Form V) from Summative Posttest Form I. This was accomplished by the simple reordering of the test items within each of the four "sections" of the summative posttest. For example, test items 1 through 8 on Summative Posttest Form I could be viewed as comprising a "section" of problems related to the addition of directed numbers. The corresponding "section" on Summative Posttest Form III, therefore, consisted of exactly the same test items but rearranged in a different order. Such was also the case for the various test items which comprised the "sections" on subtraction, multiplication, and division. Identical procedures were taken in the "construction" of Summative Posttest Form V.

In summation, Forms I, III, and V of the summative posttest are in reality only one form, Form I. Summative Posttest Form III and Summative Posttest form V, therefore, are of a pseudo nature. With respect to the second authentic form of the summative posttest, namely, Summative Posttest Form II, procedures identical to those cited in the preceding paragraph were followed in the "construction" of Form IV and Form VI. Hence, Forms II, IV, and VI of the summative posttest are in reality only one form, Form II. Summative Posttest Form IV and Summative Posttest Form VI, therefore, are of a pseudo nature.

Finally, it must be pointed out that in addition to the researcher's concern for the security of the summative posttest, the procedures described above were adopted also out of a concern for the potential threat to the reliability of the instrument that would have accompanied the development and implementation of three or more authentically distinctive forms.

Content validity. The issue of content validity relative to the two forms of the summative posttest was approached in a manner identical to the procedures used in demonstrating the content validity of the formative trial tests. More specifically, the assumption was made that the content validity of Summative Posttest Form I and Summative Posttest Form II was ensured due to the direct correspondence between the various test items and the performance objectives upon which they were based relative to subject-matter content, required student behavior, and the condition under which the behavior had to be demonstrated. Furthermore, this assumption was corroborated by the professional judgment of the three certified mathematics teachers whose students comprised the sample in this study.

Reliability coefficients obtained in a pilot setting. Prior to the commencement of the study on September 24, 1973, both forms of the summative posttest were administered in a pilot setting for the purpose of investigating the following technical characteristics of the instrument: parallel forms reliability (by way of Pearson product-moment coefficient), internal consistency reliability (by way of Kuder-Richardson formula 20 coefficient), standard deviation, and standard error of measurement.

The pilot setting for the initial administration of Summative Posttest Form I and Summative Posttest Form II was in a location other than the site of the investigation itself and involved an Algebra I class of 31 male students who had just recently completed a unit of work on the addition, subtraction, multiplication, and division of directed numbers. The school in which the initial testing occurred was organized on a conventional, graded basis; furthermore, all mathematics classes were of a homogeneous nature. The particular class involved in the pilot testing was composed of average to slightly below average students relative to mathematics placement.

Summative Posttest Form I was administered to the class of 31 students by the regular classroom teacher with the understanding that the results were to be used for the purpose of assessing their achievement on the unit of work just completed. Exactly one week later Summative Posttest Form II was administered with the understanding that a higher score on the second form would replace the earlier score attained on the first form but that a lower score on the second form would not supersede or be averaged with the previous score.

The parallel forms reliability coefficient for Summative Posttest Form I and Summative Posttest Form II was .89. For Summative Posttest Form I the internal consistency reliability coefficient was .68 with a standard deviation and a standard error of measurement equal to 3.51 and 1.98, respectively. For Summative Posttest Form II the internal consistency reliability coefficient was .84 with a standard deviation and a standard error of measurement equal to 4.68 and 1.88, respectively.

The data just cited, as well as any data reported in this study relative to student achievement on the summative posttest, are based upon a possible range of raw scores extending from 0 to 25.

Reliability coefficients and item analysis data obtained from the research sample. The purpose of this section is to report findings relative to the following technical characteristics of the two forms of the summative posttest: internal consistency reliability, standard error of measurement, item difficulty levels, and point-biserial coefficients between individual items and total test score. In order to accomplish this task, it was necessary to analyze Forms III and V as manifestations of Form I. In a similar fashion, Forms IV and VI were analyzed as manifestations of Form II. Hence, any further reference made in this study to Summative Posttest Form I is to be interpreted as being inclusive of Summative Posttest Form III and Summative Posttest Form V. Likewise, any further reference made in this study to Summative Posttest Form II is intended as being inclusive of Summative Posttest Form IV and Summative Posttest Form VI.

As indicated in Table 1, the internal consistency reliability coefficient of Summative Posttest Form I for the combination of the mastery learning and the control groups was .86 with a standard deviation and a standard error of measurement equal to 5.28 and 1.96, respectively. The reliability coefficient for the mastery learning group was .77 with a standard deviation and a standard error of measurement equal to 3.92 and 1.89, respectively. The reliability coefficient for the control group was .85 with a standard deviation and a standard error of measurement equal to 4.98 and 1.96, respectively.

Tables 2 and 3, as well as Appendix K, present item analysis data relative to Summative Posttest Form I. Table 2 summarizes the number of items associated with ten possible ranges of item difficulty indices. This information, in conjunction with data presented in Appendix K, gives evidence of the following:

1. When both groups are considered in combination, 16 of the 25 items appear in the medium range of difficulty indices (.31 to .70).

2. When only the mastery learning group is considered, 7 of the 25 items appear in the medium range of difficulty indices (.31 to .70). This is contrasted with the control group which shows 10 of the 25 items appearing in the medium range.

TABLE 1

Internal Consistency Reliability Coefficients, Standard Deviations, and Standard Errors of Measurement for Summative Posttest Form I

Group	Variables			
	n	Reliability[a]	S. D.	S. E. M.
MLG	37	.77	3.92	1.89
CG	34	.85	4.98	1.96
MLG & CG	71	.86	5.28	1.96

[a] Using Kuder-Richardson formula 20.

TABLE 2

Item Frequencies According to Ranges of Item Difficulty Indices for Summative Posttest Form I

Group	Ranges of Item Difficulty Indices										Total
	.00-.10	.11-.20	.21-.30	.31-.40	.41-.50	.51-.60	.61-.70	.71-.80	.81-.90	.91-1.00	
MLG	0	0	0	1	0	4	2	5	9	4	25
CG	0	3	4	2	2	2	4	3	4	1	25
MLG & CG	0	0	0	5	2	2	6	3	6	1	25

3. When both groups are considered in combination, 50 percent or more of the students answered correctly 18 of the 25 items.

4. When both groups are not considered in combination, 50 percent or more of the mastery learning group responded correctly to 24 of the 25 items while the same percentage of the control group answered correctly 15 of the 25 items.

Contained in Table 3 is an account of the number of items associated with ten possible ranges of item discrimination indices. This information, in conjunction with the point-biserial correlation coefficients presented in Appendix K, provides the following evidence:

1. When both groups are considered in combination, 24 of the 25 items have point-biserial correlation coefficients greater than .30.

2. When only the mastery learning group is considered, 22 of the 25 items have point-biserial correlation coefficients greater than .30. In the control group, 21 of the 25 items have point-biserial correlation coefficients greater than .30.

3. In all instances every item was of a positively discriminating nature.

As indicated in Table 4, the internal consistency reliability coefficient of Summative Posttest Form II for the combination of the mastery learning and the control groups was .86 with a standard deviation and a standard error of measurement equal to 5.41 and 2.03, respectively.

TABLE 3

Item Frequencies According to Ranges of Item Discrimination Indices
(Point-biserial Correlation Coefficients Between Individual
Test Items and Total Test Score) for
Summative Posttest Form I

Group	Ranges of Item Discrimination Indices (r_{pbi})										Total
	.00-.10	.11-.20	.21-.30	.31-.40	.41-.50	.51-.60	.61-.70	.71-.80	.81-.90	.91-1.00	
MLG	0	2	1	10	8	1	3	0	0	0	25
CG	0	0	4	1	9	9	2	0	0	0	25
MLG & CG	0	0	1	3	12	6	3	0	0	0	25

TABLE 4

Internal Consistency Reliability Coefficients, Standard Deviations, and Standard Errors of Measurement for Summative Posttest Form II

Group	Variables			
	n	Reliability[a]	S. D.	S. E. M.
MLG	27	.75	3.88	1.92
CG	43	.83	4.97	2.04
MLG & CG	70	.86	5.41	2.03

[a] Using Kuder-Richardson formula 20.

The reliability coefficient for the mastery learning group was .75 with a standard deviation and a standard error of measurement equal to 3.88 and 1.92, respectively. The reliability coefficient for the control group was .83 with a standard deviation and a standard error of measurement equal to 4.97 and 2.04, respectively.

Tables 5 and 6, as well as Appendix L, present item analysis data relative to Summative Posttest Form II. Table 5 summarizes the number of items associated with ten possible ranges of item difficulty indices. This information, in conjunction with data presented in Appendix L, gives evidence of the following:

1. When both groups are considered in combination, 14 of the 25 items appear in the medium range of difficulty indices (.31 to .70).

2. When only the mastery learning group is considered, 10 of the 25 items appear in the medium range of difficulty indices (.31 to .70). This is contrasted with the control group which shows 13 of the 25 items appearing in the medium range.

3. When both groups are considered in combination, 50 percent or more of the students answered correctly 17 of the 25 items.

4. When both groups are not considered in combination, 50 percent or more of the mastery learning group responded correctly to 23 of the 25 items while the same percentage of the control group answered correctly 15 of the 25 items.

TABLE 5

Item Frequencies According to Ranges of Item Difficulty Indices
for Summative Posttest Form II

Group	Ranges of Item Difficulty Indices										Total
	.00-.10	.11-.20	.21-.30	.31-.40	.41-.50	.51-.60	.61-.70	.71-.80	.81-.90	.91-1.00	
MLG	0	0	0	0	2	3	5	3	11	1	25
CG	0	3	2	4	1	6	4	3	2	0	25
MLG & CG	0	0	1	4	5	0	5	5	5	0	25

Contained in Table 6 is an account of the number of items associated with ten possible ranges of item discrimination indices. This information, in conjunction with the point-biserial correlation coefficients presented in Appendix L, provides the following evidence:

1. When both groups are considered in combination, 23 of the 25 items have point-biserial correlation coefficients greater than .30.

2. When only the mastery learning group is considered, 19 of the 25 items have point-biserial correlation coefficients greater than or equal to .30. In the control group, 23 of the 25 items have point-biserial correlation coefficients greater than .30.

3. In all instances every item was of a positively discriminating nature.

Criterion-Referenced Measurement

The formative trial tests and the summative posttest described above were of a criterion-referenced nature in that their purpose was to assess the performance of each student relative to a predetermined criterion. This was in contrast with a norm-referenced testing situation in which the purpose is to assess the performance of each student relative to his comparative position within the group. The objective of this section is twofold: (*a*) to acknowledge the criterion-referenced orientation of the procedure identified earlier for the construction of the formative trial tests and the summative posttest and (*b*) to reflect from a criterion-

TABLE 6

Item Frequencies According to Ranges of Item Discrimination Indices
(Point-biserial Correlation Coefficients Between Individual
Test Items and Total Test Score) for
Summative Posttest Form II

Group	Ranges of Item Discrimination Indices (r_{pbi})										Total
	.00-.10	.11-.20	.21-.30	.31-.40	.41-.50	.51-.60	.61-.70	.71-.80	.81-.90	.91-1.00	
MLG	3	2	5	3	4	6	1	0	1	0	25
CG	1	1	0	5	9	7	2	0	0	0	25
MLG & CG	1	1	0	2	6	12	2	1	0	0	25

referenced perspective upon the reliability coefficients and item analysis data reported in preceding sections.

Test construction. The procedure identified by Popham & Husek (1969) for the construction of criterion-referenced test items was adhered to in the development of the formative trial tests and the summative posttest. More specifically, the chief concern in the construction of the instruments was to ensure that each test item accurately reflected the desired criterion behavior. The selection of an item for inclusion on an instrument was not based upon the extent to which it represented an item of "average difficulty" as opposed to one of "extreme easiness" or "extreme difficulty." In other words, the intention of the researcher was not to include or to exclude an item solely because of the anticipated degree to which it would produce variability among the students. Regardless of its apparent ease or difficulty, an item was deemed acceptable if it represented the class of behaviors defined by the criterion.

Consistent with the discussion immediately above and relevant to the use made of the summative posttest in this study is the assertion by Glaser (1963) that

> . . . achievement tests used primarily to provide information about differences in treatments need to be constructed so as to maximize the discriminations made between groups treated differently and to minimize the differences between the individuals in any one group. Such a test will be sensitive to the differences produced by instructional conditions. For example, a test designed to demonstrate the effectiveness of instruction would be constructed so that it was generally difficult for those taking it before training and generally easy after training. The content of the test used to

> differentiate treatments should be maximally sensitive to the performance changes anticipated from the instructional treatments (p. 520).

Reliability. As alluded to earlier, test items on a criterion-referenced instrument are written for the purpose of assessing student performance relative to established criteria. Furthermore, one possible application of such items is to maximize discriminations between instructional treatments, a function in direct opposition to the use made of norm-referenced items in maximizing discriminations among students within a group.

With respect to determining the reliability of criterion-referenced tests, Popham & Husek (1969) made the following observations: Since the use of test items to discriminate among individuals within a group is not consistent with a criterion-referenced instrument, the appeal to classical procedures for determining the reliability of such a test, procedures that are contingent upon variability of scores within a group, is not totally appropriate. Unfortunately, alternative procedures have not yet been identified. The point that must be kept in mind, however, is not that conventional reliability indices cannot be used to support the consistency of a test but rather that ". . . a criterion-referenced test could be highly consistent, either internally or temporarily, and yet indices dependent on variability might not reflect that consistency (pp. 5-6)."

Hence, the reliability coefficients reported earlier on Summative Posttest Form I and Summative Posttest Form II,

.86 and .86, respectively, can be viewed as evidence which supports the internal consistency of the two forms.

Item analysis data. Based upon what has already been indicated relative to a criterion-referenced instrument's independence of score variability among individuals within a group, one is inclined to question the value of reporting item analysis data such as indices of item difficulty and item discrimination.

With respect to the use of item difficulty indices, comparisons can be made between treatment groups relative to the number of items answered correctly by a given proportion of students. Furthermore, when the data from both groups are considered in combination, such indices can be utilized to ascertain the degree of rigor demonstrated by the instrument. Both of these concerns have been addressed in a previous presentation and discussion of the data contained in Tables 2 and 5 and Appendices K and L.

With respect to the use of item discrimination indices, Popham & Husek (1969) made the following assertions: Standards conventionally associated with the discrimination indices of norm-referenced items can be relaxed in a criterion-referenced situation. An item which fails to discriminate need not be viewed as a poor item provided that it reflects the desired criterion behavior. In terms of a positively discriminating item, just as much respectability should be afforded it in a criterion-referenced test as is done in a norm-referenced instrument. Regarding a negatively discriminating item, the need for suspicion and careful analysis is just as real in a criterion-referenced setting as it is in a norm-referenced situation.

As already indicated in an earlier summarization of the data contained in Tables 3 and 6 and Appendices K and L, every item that appeared on either form of the summative posttest was of a positively discriminating nature, with the vast majority exceeding a point-biserial correlation coefficient of .30.

Assessment of Perseverance

Construction of instructional material and test item.

An instrument referred to as Assessment of Perseverance (see Appendix H) was developed. It was intended that this instrument be administered to each student for the purpose of determining the amount of time he would be willing to spend actively engaged with new instructional material and a single test item associated with the same. The data obtained by the Assessment of Perseverance are considered a representative measure of the perseverance variable as defined by Carroll and, hence, should not be confused with the rough approximation of this variable as represented by the number of classes spent by each student. With respect to the actual construction of the Assessment of Perseverance, the following steps were implemented:

1. It was recognized that the second part of Learning Sequence 332 (with which the study was not directly concerned)

would require the student to apply his knowledge of the four basic operations to the solution of open sentences.

2. The decision was made to provide each student with a one-page explanation of how to solve an open sentence that required the use of either the multiplication or division property of equality. As far as could be ascertained, this brief set of instructional material represented the first exposure of each student to the aforementioned topic.

3. Having thus prepared the explanatory material which was to introduce the new topic, it was necessary to construct a single test item which would confront the student with the task of solving an open sentence. More specifically, an equation was constructed which required that both the multiplication and division properties of equality be used in its solution. Furthermore, the equation was constructed in such a way that the student would first have to simplify the numerator and denominator on both sides of the equation prior to being able to recognize it as a rather simple open sentence. It was intended that the development and use of this type of test item would require the student to apply not only his newly acquired knowledge of solving open sentences but also his knowledge of the four basic operations.

4. The combination of the instructional material and the single test item was judged as being sufficiently representative of a difficult learning task. The complexity of the task was assumed to be such that mastery of it would be just beyond attainment by practically all students in the

sample. (The mastery of the task by only one student out of a total of 141 corroborated the aforementioned assumption.) It was reasoned that only by the presentation of an extremely challenging task could a valid measure of perseverance be achieved.

5. Due to the high complexity level of the single test item and the assumed inability of the vast majority of students to master it, only one form of the instrument was deemed necessary.

Construct validity. The instrument just described was used for the purpose of obtaining data thought to be representative of the variable identified by Carroll as perseverance. Due to the manner in which this variable is defined as well as the unavailability of a second instrument which purported to measure perseverance, the construct validity of the instrument labeled Assessment of Perseverance was assumed rather than demonstrated empirically.

Data Collection Procedures

California Short-Form Test of Mental Maturity, 1963 Revision

In order to obtain for each student a measure of ability to understand instruction consistent with Carroll's definition of this variable, the California Short-Form Test of Mental Maturity, 1963 Revision, was administered to the 169 students who comprised the initial sample. Each intelligence quotient score which resulted represented a composite of a language subscore and a non-language subscore.

Due to the size of the sample, two group-testing sessions were conducted by the researcher with the assistance of two proctors. The first session, which occurred on Tuesday, September 11, 1973, involved a total of 68 students; the second session, which took place the following day, encompassed the remaining 101 students. In both instances the mid-morning part of the school day was used. Although the group testing sessions spanned a period of two days, no unusual events occurred on either day which might have jeopardized the accuracy of the resulting scores.

Summative Posttest

The various forms of the summative posttest were administered for the purpose of assessing the achievement of students relative to the eight performance objectives specified in their instructional packet. In the mastery learning group, the summative posttest was administered to a student during the mathematics class immediately subsequent to his successful completion of Formative Trial Test II and the necessary learning correctives (if any were prescribed). In the control group, the summative posttest was administered to a student during the mathematics class immediately subsequent to his successful completion of all the assignments which corresponded to performance objectives 1 through 8.

The assignment of the various forms of the summative posttest to given students was made in sequential order (that is, Form I through Form VI). The actual administration of the summative posttest was conducted in a regular-sized classroom designated as the "testing room" and was supervised by paraprofessionals. As mentioned earlier, each form of the summative posttest consisted of 25 test items; hence, raw data which resulted from the scoring of each test had a possible range extending from 0 to 25. For the purposes of this study, only the actual raw scores were considered; no attempt was made by the researcher to assign grades to the students, although this was done by the regular teacher.

Assessment of Perseverance

The instrument labeled Assessment of Perseverance was intended to measure the amount of time a student would be willing to spend actively engaged with a difficult learning task. Each student in the sample was administered this instrument during the mathematics class immediately subsequent to his completion of the summative posttest.

In each instance the purpose of the exercise as explained on the cover page and the directions as stated at the top of the second page (see Appendix H) were read to the student prior to his actual commencement of work on the learning task. Due to the obvious role played by the instructional material and test item in terms of introducing the student to a forthcoming topic, no mention was made relative to the researcher's ultimate purpose behind the administration of this instrument. Furthermore, no questions were posed by any of the students relative to the title "Assessment of Perseverance." Though not included in the instrument itself, directions were given to each student relative to the fact that his performance on the test item would not be used for grading purposes.

The act of administering the Assessment of Perseverance was conducted by the researcher himself. The actual type of data obtained was in terms of the total number of minutes and seconds spent by each student (as measured by a stopwatch) while engaged with the instructional material and the test item. The time devoted to the reading of the statement of purpose and the directions was not included. Due to the staggered fashion in which the students completed their summative posttest, this aspect of the study never involved a group of more than five students per class session.

Tabulation of Classes Spent

For the duration of the study, an accurate account was maintained of the total number of instructional periods in algebra attended by each student while completing the designated unit of instruction. This record was amassed by way of a simple attendance check on those students who were involved with part one of Learning Sequence 332 at a given time. Excluded from this tabulation were the two class sessions devoted by each student to the completion of the summative posttest and the Assessment of Perseverance. Included in this tabulation, though, were the number of class sessions spent by each member of the mastery learning group in completing the two formative trial tests.

Appendix I includes the form entitled "Record of Classes Spent by Student" that was used in the study: Appendix J encompasses the form labeled "Record of Student Activity and Performance" that served as a summarization sheet for all pertinent data collected during the duration of the investigation.

Research Design

The experimental design employed in the implementation of this study can be characterized as a logical extension and concurrent replication of the Posttest-Only Control Group Design presented by Campbell & Stanley (1963). A symbolic representation of the design used in this study follows:

HAb	R	E	O
	R	C	O
AAb	R	E	O
	R	C	O
LAb	R	E	O
	R	C	O

The HAb, AAb, and LAb symbolize the high-, average-, and low-ability levels into which the initial sample of 169 students was divided. The E and the C represent the experimental (mastery learning) group and the control group, respectively. The R's in the second column indicate the random assignment of students in each ability level to the two treatments. Finally, the O's in the fourth column denote the observed measures for each group relative to the summative posttest, the Assessment of Perseverance, and the tabulation of classes spent.

The crossing of the three levels of ability to understand instruction with the two levels of treatment resulted in a 3 X 2 fixed-effects factorial design. This factorial design, therefore, made possible the investigation of the interaction between ability to understand instruction and quality of instruction relative to the three dependent variables.

As might be inferred from the illustration and discussion of the design immediately above, a basic assumption of the study was the contention that any observed differences between the mastery learning group and the control group relative to the dependent variables were a result of the differences in the quality of instruction of each treatment. Hence, it was assumed that the remaining variables of the model, aptitude, ability to understand instruction, opportunity, and perseverance (when not used as a dependent variable), did not differ between the two groups and, hence, were not responsible for any significant differences between the two treatments relative to degree of learning. When perseverance (classes spent in completing the designated unit of instruction and minutes of active engagement on a difficult learning task) was investigated as a dependent variable, it was hypothesized to differ significantly between groups due to the differences in the quality of instruction of each group. In effect, then, the essence of this entire paragraph is a restatement of the law of the single variable or the method of difference as defined by Mill (1873). Table 7 provides a summary of the various bases used to substantiate the aforementioned assumptions.

Data Analysis Procedures

Statistical techniques that were employed to analyze the data in this study included the following: two-way fixed-effects analysis of variance (unweighted means), two-way fixed-effects analysis of covariance (unweighted means), Pearson product-moment correlation coefficient, partial correlation coefficient, and Fisher's Z-transformation of r. Furthermore, the Scheffe test for multiple comparisons was used to investigate significant main effects of ability to understand instruction. Also, post hoc comparisons for interaction as developed by Marascuilo & Levin (1970) were utilized to explore significant interaction effects of ability to understand instruction with quality of instruction.

The correspondence between the various statistical procedures just cited and the research hypotheses to which they were applied was as follows: two-way fixed-effects

TABLE 7

Assumptions and Bases Underlying the Application of Mill's Law of the Single Variable to the Present Study

Assumptions	Bases
1. The groups did differ in quality of instruction.	1. By definition
2. The groups did not differ in aptitude.	2. Randomization
3. The groups did not differ in ability to understand instruction.	3. Empirical evidence (the leveling of each treatment in accordance with three ranges of intelligence quotient scores)
4. The groups did not differ in opportunity.	4. Unlimited time allotted
5. The groups did not differ in perseverance (classes spent) when degree of learning was investigated as a dependent variable.	5. Randomization and/or statistical adjustments (number of classes spent used as a covariate)
6. The groups did differ in perseverance (classes spent in completing the designated unit of instruction and minutes of active engagement on a difficult learning task) when this variable was investigated as a dependent variable.	6. As hypothesized

analysis of variance (unweighted means)--research hypotheses 1, 2, 7, 8, 11, and 12; two-way fixed-effects analysis of covariance (unweighted means)--research hypotheses 4, 5, 14, and 15; Pearson product-moment correlation coefficient--research hypotheses 3, 9, 10, 13, and 17; partial correlation coefficient--research hypotheses 6 and 16; Fisher's Z-transformation of r--research hypotheses 3, 6, 9, 10, 13, 16, and 17; Scheffé's test for multiple comparisons--research hypotheses 1 and 4; Marascuilo & Levin's (1970) post hoc comparisons for interaction--research hypotheses 2 and 5. The testing of the null form of each of the research hypotheses was conducted at the .05 level of significance.

Consideration was given to the basic assumptions underlying analysis of variance: (_a_) additive nature of the component contributions to the total variance; (_b_) random sampling to ensure independent observations within groups; (_c_) normally distributed population values within groups; and (_d_) homogeneity of variances within groups. According to Glass, Peckham, & Sanders (1972), whether or not the additivity assumption is violated should be of little concern to the researcher. With respect to the second assumption, the randomization and the design used in this study served as an appropriate safeguard. Concerning the third and fourth assumptions, reliance was placed upon the contention made by Dayton (1970) that ". . . analysis of variance is virtually unaffected by violations of normality and homogeneity of variance _if_ the samples entering into the analysis are of the same, or approximately the same, size (p. 35)."

Attention was devoted also to the basic assumptions underlying analysis of covariance: (a) randomization, (b) normality, (c) homogeneity of variances, (d) homogeneity of regression, (e) linearity, (f) covariate measured without error, and (g) covariate independent of treatment. As was the case with analysis of variance, the assumption of random sampling to ensure independent observations was met by virtue of the randomization and the design used in this study. With respect to the assumptions of normality and homogeneity of variances, Cochran (1957) and Winer (1962) have indicated that the robustness of analysis of variance to violations of these two assumptions extend into analysis of covariance. Concerning the assumption of homogeneity of regression, Glass et al. (1972) contended that ". . . it appears that one is not very likely to make Type I errors due to heterogeneity of regression slopes alone (p. 277)." Regarding the necessity for a linear relationship between the dependent variable and the covariate, scatter plots for each treatment group gave evidence that this assumption was met.

The analysis of covariance assumptions which demand that the covariate be measured without error and that the covariate be independent of treatment necessitate a special discussion. Concerning the former assumption, the variables classes spent and degree of learning were used as covariates. The measurement of classes spent involved simply the review of student attendance records and the tabulation of the number of pertinent instructional periods attended. Obviously, the accuracy of this type of measurement is easily corroborated by an independent observer. With respect to the measurement of degree of learning, both forms of the summative posttest gave evidence of acceptable parallel forms and internal consistency reliability coefficients.

Regarding the rationale behind the assumption that the covariate is independent of treatment, Elashoff (1969) made the following observation: "When the covariate . . . is affected by the treatment, the regression adjustments may remove part of the treatment effect or produce a spurious treatment effect (p. 388)." This assumption was not met relative to either classes spent or degree of learning. However, though admittedly unorthodox, it is the contention of this researcher that the dependence of the two covariates upon treatment did not render the use of analysis of covariance inappropriate but rather served to accommodate a specific type of investigation undertaken in this study. In order to illustrate this point, a description follows of the objective underlying the designation of degree of learning as a covariate when perseverance was investigated as a dependent variable: The third assumption pertinent to this study as specified in Chapter 1 demonstrated the indirect influence of treatment upon perseverance by asserting that a high quality of instruction implies a high degree of learning which in turn implies a large amount of perseverance. If one is interested in investigating the direct impact of treatment upon perseverance with the intermediate step involving degree of learning removed, then the situation described earlier by Elashoff is exactly what is needed. Hence, the removal of that part of the treatment related to degree of learning was desirable in order to assess the direct effect of treatment (though modified) upon perseverance. The justification behind the use of analysis of covariance to investigate degree of learning as a dependent variable with classes spent as the covariate followed the same argument as that just cited.

Limitations

The various limitations inherent in this study included the following:

1. The reactive effects of experimental procedures upon students represented a threat to the external validity of the study. More specifically, the formative trial tests and the learning correctives to which the mastery learning group was subjected served to emphasize to those students the distinctive treatment received by them. Although the members of both groups were integrated in all three of the class periods designated in this study, it is safe to assume that the students in the mastery learning group were eminently aware of the special tests and materials prepared for them. Hence, it is feasible that their reactions to the various learning tasks were not only a result of the treatment per se but also a consequence of their having been singled out for special attention.

2. Due to the relatively long duration of the investigation--September 24, 1973 to December 7, 1973--that threat

to the internal validity of the study known as contemporary history was present. Although the number of classes spent by the various students in completing the designated work ranged only from 2 to 28, the staggered manner in which the students entered the study resulted in the collection of dependent-variable data in an equally staggered fashion. For example, some students completed their summative post-test and Assessment of Perseverance in the last week of September while others did not complete the same exercises until the first week of December. It is suggested, then, that the vast discrepancies among students relative to the actual time at which their dependent-variable measures were made might very well have contributed to performances which were either hampered or enhanced by specific events.

3. A total of 28 students were lost to the study as a result of experimental mortality. This threat to the internal validity of the investigation is especially important when one considers the fact that 21 of the 28 students that were eliminated from the study had been assigned previously to the mastery learning group. Certainly the retention of these students in the mastery learning group would have contributed to a more accurate assessment of the impact of a mastery learning treatment upon the designated dependent variables.

4. As alluded to earlier, the dependent variable designated as classes spent is only a rough approximation of the variable identified by Carroll as perseverance. Quite obviously, mere attendance during a particular class period

was not necessarily indicative of time actively spent on a learning task. Hence, the data collected on this extremely rough measure of perseverance should be interpreted accordingly.

5. The Assessment of Perseverance instrument always was administered during a given class session to either a single individual or a group not in excess of five students. A measurements limitation of this study relative to perseverance as a dependent variable encompassed the fact that students who were tested individually were not subjected to the peer-group influence which was a part of those situations in which a collection of two to five students were involved. In this latter instance, it is assumed that the extent of perseverance by any given student or combination of students served to contribute to the extent of perseverance displayed by the remaining students. Restrictions in terms of the availability of testing areas and time precluded the constant provision of an individual, one-on-one situation for assessing the perseverance of students.

CHAPTER 4

Presentation and Discussion of the Findings

Presentation of the Findings

That aspect of the research design which involved the crossing of the three ability levels with the two treatment levels was intended to reduce the experimental error involved in testing for the main effects of treatment and the interaction effects of ability to understand instruction and quality of instruction relative to the three dependent variables. Though not hypothesized in this study, significant main effects of ability to understand instruction relative to the three dependent variables were expected due to the relationship between each of the three dependent variables and ability to understand instruction.

Hypotheses Related to Degree of Learning

Preface to hypotheses 1 through 3. Table 8 presents the cell summary for the unweighted means analysis of variance of achievement scores for both treatment groups crossed with the three ability levels. The row or ability level means were 17.77, 14.47, and 13.26 for the high-, average-, and low-ability levels, respectively. Table 9 presents the summary table for the unweighted means analysis of variance of achievement scores for both treatment groups crossed with the three ability levels. The F ratio for the

TABLE 8

Cell Summary for Unweighted Means Analysis of Variance of Achievement Scores for Mastery Learning and Control Groups Crossed With Three Ability Levels

Factor A: Ability	Factor B: Treatments					
	MLG		CG		Rows	
	n	Mean	n	Mean	n	Mean
High	27	20.37	33	15.64	60	17.77
Average	26	17.73	33	11.91	59	14.47
Low	11	18.00	11	8.73	22	13.36
Columns	64	18.89	77	13.05		

TABLE 9

Unweighted Means Analysis of Variance of Achievement Scores for Mastery Learning and Control Groups Crossed With Three Ability Levels

Source of Variance	Degrees of Freedom	Sum of Squares	Mean Squares	F
Ability Levels	2	425.04	212.52	12.42**
Treatments	1	1236.67	1236.67	72.27**
Ability Levels X Treatments	2	105.98	52.99	3.10*
Error	135	2309.96	17.11	
Total	140	4077.64		

*Significant at the .05 level.
**Significant at the .001 level.

main effects of ability to understand instruction equaled 12.42 and was significant at the .001 level.

Table 10 presents the Scheffe method of multiple comparisons as applied to the achievement scores for the three ability levels. The contrast between the high-ability and average-ability levels and the contrast between the high-ability and low-ability levels were both significant at the .001 level. Based upon the comparisons among the row or ability level means cited earlier, this significant finding favored the high-ability level. The contrast between the average-ability and low-ability levels, however, was not of a significant nature. Hence, significant findings pertinent to two of the three possible contrasts involving ability levels resulted.

Hypothesis 1. The first question to which an answer was sought in this study asked how quality of instruction would affect degree of learning in a setting of unlimited opportunity. Research hypothesis 1 predicted that there would be a significant difference between the mastery learning group and the control group relative to degree of learning. More specifically, it was predicted that the mastery learning group would attain a significantly greater degree of learning than would the control group. Stated in null form, this hypothesis can be represented as follows:

$$\text{Ho1: } MAc_{MLG} - MAc_{CG} = 0$$

where MAc_{MLG} and MAc_{CG} equal the mean achievement scores on a summative posttest for the mastery learning group and the control group, respectively.

TABLE 10

Scheffé Method of Multiple Comparisons As Applied to Achievement Scores for the Three Ability Levels

Contrast	Estimate of Contrast	Estimate of Variance of Contrast	Estimate of Standard Deviation of Contrast	Ratio of Estimate of Contrast to Estimate of Standard Deviation of Contrast
$u_{1.} - u_{2.}$	3.30	.57	.76	4.35*
$u_{1.} - u_{3.}$	4.41	1.06	1.03	4.28*
$u_{2.} - u_{3.}$	1.11	1.07	1.03	1.08

*Significant at the .001 level.

As presented in Table 8, the column or treatment means were 18.89 for the mastery learning group and 13.05 for the control group. According to Table 9, the F ratio for the main effects of treatment equaled 72.27; hence, the null hypothesis was rejected at the .001 level of significance. As evidenced by the aforementioned comparison between column or treatment means, the prediction of the experimental consequence which accompanied research hypothesis 1 was accurate in that $MAc_{MLG} > MAc_{CG}$.

Hypothesis 2. The second question to which an answer was sought in this study asked what the interaction would be between ability to understand instruction and quality of instruction relative to degree of learning in a setting of unlimited opportunity. Research hypothesis 2 predicted that there would be a significant ordinal interaction between ability to understand instruction and quality of instruction relative to degree of learning. More specifically, it was predicted that as students decrease in ability to understand instruction, their degree of learning would decrease in both the mastery learning group and the control group; however, the extent of decrease would be significantly greater in the control group than in the mastery learning group. Stated in null form, this hypothesis can be represented as follows:

$$Ho2: MAc_{MLG,LAb} - MAc_{CG,LAb} = MAc_{MLG,AAb} - MAc_{CG,AAb} = MAc_{MLG,HAb} - MAc_{CG,HAb}$$

where $MAc_{MLG,LAb}$, $MAc_{MLG,AAb}$, and $MAc_{MLG,HAb}$ equal the mean achievement scores on a summative posttest for the mastery learning group which is divided into low-, average-, and high-ability levels, respectively, and $MAc_{CG,LAb}$, $MAc_{CG,AAb}$, and $MAc_{CG,HAb}$ equal the mean achievement scores on a summative posttest for the control group which is divided into low-, average-, and high-ability levels, respectively.

As indicated in Table 9, the F ratio for the interaction effects of ability to understand instruction and treatment equaled 3.10; hence, the null hypothesis was rejected at the .05 level of significance. Figure 6 illustrates the graph of the significant ordinal interaction found between ability to understand instruction and quality of instruction relative to achievement scores.

For the purpose of ensuring an accurate description of the combination of cells which contributed to the significant ordinal interaction, Table 11 presents the post hoc comparisons of the differences between the differences in cell means. As can be seen from this table, the tetrad difference involving the high- and low-ability levels was significant at the .05 level. The four cells involved in this particular tetrad difference, therefore, combined to effect the significant ordinal interaction between ability to understand instruction and quality of instruction. The tetrad differences involving the high- and average-ability levels, as well as the average- and low-ability levels, were not of a significant nature and, hence, did not encompass cells which contributed to the significant ordinal interaction. The prediction of the experimental consequence which accompanied research hypothesis 2,

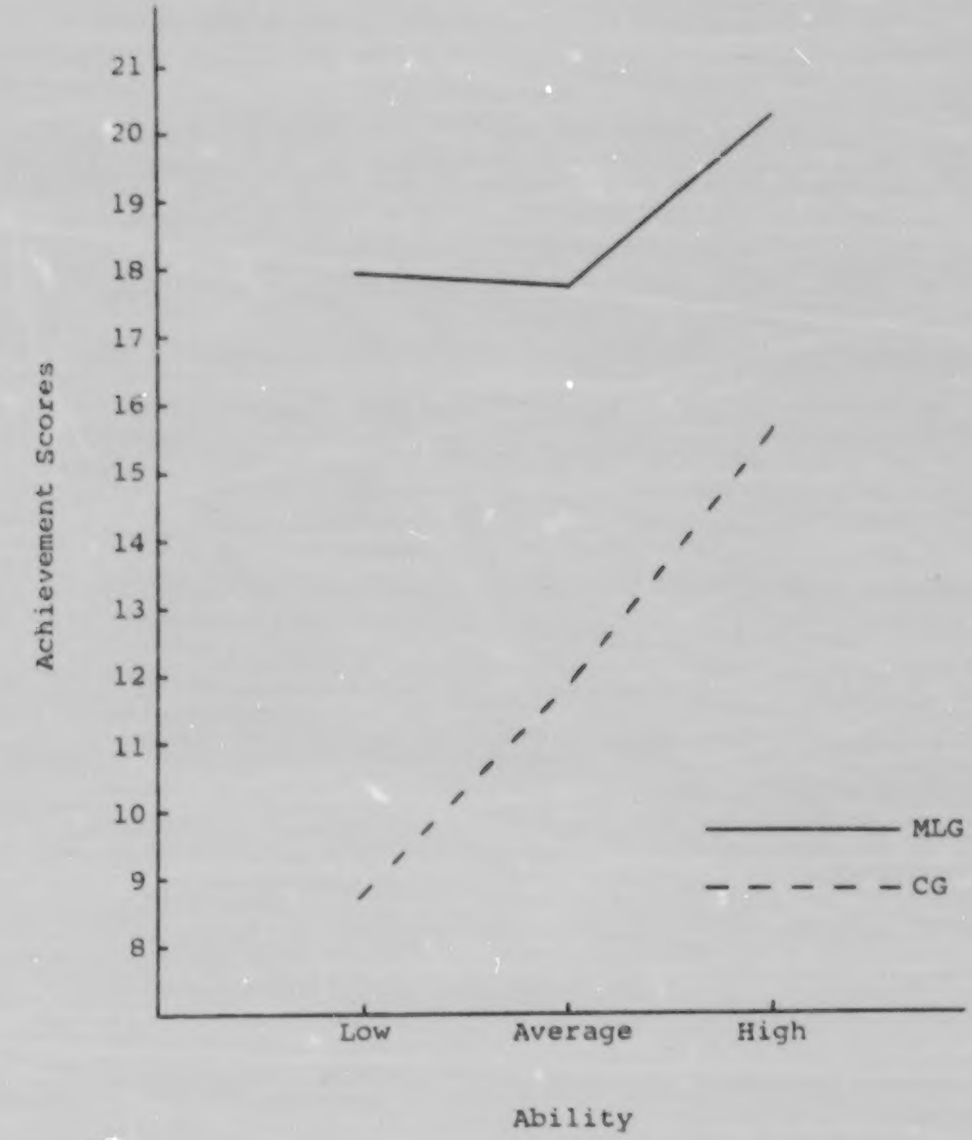

Fig. 6. Illustration of the significant ordinal interaction found between ability to understand instruction and quality of instruction relative to achievement scores.

TABLE 11

Post Hoc Comparisons for Significant Ordinal Interaction Discovered Between Ability To Understand Instruction and Quality of Instruction Relative to Achievement Scores

Tetrad Difference[a]	Scheffé Coefficient	Standard Error of Tetrad Difference	Critical Value	Estimate of Tetrad Difference
$\triangle_1 - \triangle_2$	6.00	2.91	±4.18	1.09
$\triangle_1 - \triangle_3$	6.00	2.91	±4.18	4.54*
$\triangle_2 - \triangle_3$	6.00	2.91	±4.18	3.45

[a] $\triangle_1 = MAc_{MLG,HAb} - MAc_{CG,HAb} = 4.73$

$\triangle_2 = MAc_{MLG,AAb} - MAc_{CG,AAb} = 5.82$

$\triangle_3 = MAc_{MLG,LAb} - MAc_{CG,LAb} = 9.27$

*Significant at the .05 level.

though, was accurate in that $MAc_{MLG,LAb} - MAc_{CG,LAb} > MAc_{MLG,AAb} - MAc_{CG,AAb} > MAc_{MLG,HAb} - MAc_{CG,HAb}$.

Hypothesis 3. The third question to which an answer was sought in this study asked what the correlation would be between degree of learning and ability to understand instruction in a setting of unlimited opportunity while under a high quality of instruction or a low quality of instruction. Research hypothesis 3 predicted that there would be a significant difference between the mastery learning group and the control group relative to the correlation between degree of learning and ability to understand instruction. More specifically, it was predicted that the correlation between degree of learning and ability to understand instruction would not deviate significantly from zero in the mastery learning group but would be significantly positive in the control group, and these correlations would differ significantly from each other. Stated in null form, this hypothesis can be represented as follows:

Ho3: 1. $r_{AcAb,MLG} = 0$
2. $r_{AcAb,CG} = 0$
3. $r_{AcAb,CG} - r_{AcAb,MLG} = 0$

where $r_{AcAb,MLG}$ and $r_{AcAb,CG}$ equal the correlations between achievement scores on a summative posttest and ability for the mastery learning group and the control group, respectively.

Table 12 presents the coefficients of correlation between achievement scores and intelligence quotient scores as well as the standard deviations for both treatment groups. The correlation coefficient for the mastery learning group equaled .29 and was significant at the .025 level; hence, the first part of the null hypothesis was rejected. Contrary to the prediction of the first experimental consequence which accompanied research hypothesis 3, it was found that $r_{AcAb,MLG} \nsim 0$.

The correlation coefficient for the control group equaled .48 and was significant at the .01 level; hence, the second part of the null hypothesis was rejected. This finding was in accordance with the prediction of the second experimental consequence which accompanied research hypothesis 3 in that $r_{AcAb,CG} > 0$.

Table 13 presents the test for a significant difference between treatment groups relative to the coefficients of correlation between achievement scores and intelligence quotient scores. Fisher's Z-transformation of the correlation coefficients .29 and .48 produced Z_r's of .30 and .52 for the mastery learning group and the control group, respectively. The resulting $\bar{z}$ equaled 1.27 and was not significant; hence, the third part of the null hypothesis was not rejected. The prediction of the third experimental consequence which accompanied research hypothesis 3 was contradicted in that $r_{AcAb,CG} - r_{AcAb,MLG} \neq 0$ failed to occur at a significant level.

Preface to hypotheses 4 through 6. Table 14 presents the cell summary for the unweighted means analysis of covariance of achievement scores (with number of classes spent as

TABLE 12

Coefficients of Correlation Between Achievement Scores and Intelligence Quotient Scores for Mastery Learning and Control Groups

r		Achievement S. D.'s		Intelligence Quotient S. D.'s		N	
MLG	CG	MLG	CG	MLG	CG	MLG	CG
.29*	.48**	3.93	5.01	11.35	11.85	64	77

*Significant at the .025 level.
**Significant at the .01 level.

TABLE 13

Test for Significant Difference Between Mastery Learning and Control Groups Relative to Coefficients of Correlation Between Achievement Scores and Intelligence Quotient Scores

r		Z_r				
MLG	CG	MLG	CG	Z_r Diff.	S. E. of Z_r Diff.	$\bar{z}$
.29	.48	.30	.52	.22	.17	1.27

the covariate) for both treatment groups crossed with the three ability levels. The adjusted row or ability level means were 17.76, 14.49, and 13.33 for the high-, average-, and low-ability levels, respectively. Table 15 presents the summary table for the unweighted means analysis of covariance of achievement scores (with number of classes spent as the covariate) for both treatment groups crossed with the three ability levels. The F ratio for the main effects of ability to understand instruction equaled 13.31 and was significant at the .001 level.

Table 16 presents the Scheffe method of multiple comparisons as applied to the achievement scores (adjusted for differences in number of classes spent) for the three ability levels. The contrast between the high-ability and average-ability levels and the contrast between the high-ability and low-ability levels were both significant at the .001 level. Based upon the comparisons among the adjusted row or ability level means cited earlier, this significant finding favored the high-ability level. The contrast between the average-ability and low-ability levels, however, was not of a significant nature. Hence, significant findings pertinent to two of the three possible contrasts involving ability levels resulted.

Hypothesis 4. The fourth question to which an answer was sought in this study asked how quality of instruction would affect degree of learning in a setting of unlimited opportunity with the effects due to a rough estimate of

TABLE 14

Cell Summary for Unweighted Means Analysis of Covariance of Achievement Scores (With Number of Classes Spent As the Covariate) for Mastery Learning and Control Groups Crossed With Three Ability Levels

Factor A: Ability	Factor B: Treatments											
	MLG				CG				Rows			
		Dependent Variable		Cov.		Dependent Variable		Cov.		Dependent Variable		Cov.
	n	Adj. Mean	Unadj. Mean	Mean	n	Adj. Mean	Unadj. Mean	Mean	n	Adj. Mean	Unadj. Mean	Mean
High	27	21.04	20.37	16.33	33	15.09	15.64	9.81	60	17.76	17.77	12.40
Average	26	18.50	17.73	16.92	33	11.33	11.91	9.03	59	14.49	14.47	12.51
Low	11	18.55	18.00	15.64	11	8.12	8.73	8.82	22	13.33	13.36	12.23
Columns	64	19.57	18.89	16.45	77	12.49	13.05	9.06				

TABLE 15

Unweighted Means Analysis of Covariance of Achievement Scores (With Number of Classes Spent As the Covariate) for Mastery Learning and Control Groups Crossed With Three Ability Levels

Source of Variance	Degrees of Freedom	Sum of Squares	Mean Squares	F
Ability Levels	2	437.01	218.51	13.31**
Treatments	1	1256.87	1256.87	76.58**
Ability Levels X Treatments	2	101.45	50.72	3.09*
Error	134	2199.29	16.41	
Total	139	3994.62		

*Significant at the .05 level.
**Significant at the .001 level

TABLE 16

Scheffé Method of Multiple Comparisons As Applied to Achievement Scores
(Adjusted for Differences in Number of Classes Spent)
for the Three Ability Levels

Contrast	Estimate of Contrast	Estimate of Variance of Contrast	Estimate of Standard Deviation of Contrast	Ratio of Estimate of Contrast to Estimate of Standard Deviation of Contrast
$u_{1.} - u_{2.}$	3.27	.55	.74	4.40*
$u_{1.} - u_{3.}$	4.43	1.02	1.01	4.39*
$u_{2.} - u_{3.}$	1.16	1.02	1.01	1.15

*Significant at the .001 level.

perseverance held constant. Research hypothesis 4 predicted that with adjustments made for differences in classes spent, there would be a significant difference between the mastery learning group and the control group relative to degree of learning. More specifically, it was predicted that with adjustments made for differences in classes spent, the mastery learning group would attain a significantly greater degree of learning than would the control group. Stated in null form, this hypothesis can be presented as follows:

$$\text{Ho4: } \text{MAc(adj.)}_{MLG} - \text{MAc(adj.)}_{CG} = 0$$

where MAc(adj.)_{MLG} and MAc(adj.)_{CG} equal the mean achievement scores on a summative posttest for the mastery learning group and the control group, respectively, with adjustments made for differences in the number of classes spent in completing a given unit of instruction.

As presented in Table 14, the adjusted column or treatment means were 19.57 for the mastery learning group and 12.49 for the control group. According to Table 15, the F ratio for the main effects of treatment equaled 76.58; hence, the null hypothesis was rejected at the .001 level of significance. As evidenced by the aforementioned comparison between column or treatment means, the prediction of the experimental consequence which accompanied research hypothesis 4 was accurate in that $\text{MAc(adj.)}_{MLG} > \text{MAc(adj.)}_{CG}$.

Hypothesis 5. The fifth question to which an answer was sought in this study asked what the interaction would be between ability to understand instruction and quality of instruction relative to degree of learning in a setting of unlimited opportunity with the effects due to a rough estimate of perseverance held constant. Research hypothesis 5 predicted that with adjustments made for differences in classes spent, there would be a significant ordinal interaction between ability to understand instruction and quality of instruction relative to degree of learning. More specifically, it was predicted that with adjustments made for differences in classes spent, as students decrease in ability to understand instruction, their degree of learning would decrease in both the mastery learning group and the control group; however, the extent of decrease would be significantly greater in the control group than in the mastery learning group. Stated in null form, this hypothesis can be represented as follows:

$$\begin{aligned} \text{Ho5:}\quad & \text{MAc(adj.)}_{\text{MLG,LAb}} - \text{MAc(adj.)}_{\text{CG,LAb}} = \\ & \text{MAc(adj.)}_{\text{MLG,AAb}} - \text{MAc(adj.)}_{\text{CG,AAb}} = \\ & \text{MAc(adj.)}_{\text{MLG,HAb}} - \text{MAc(adj.)}_{\text{CG,HAb}} \end{aligned}$$

where $\text{MAc(adj.)}_{\text{MLG,LAb}}$, $\text{MAc(adj.)}_{\text{MLG,AAb}}$, and $\text{MAc(adj.)}_{\text{MLG,HAb}}$ equal the mean achievement scores on a summative posttest for the mastery learning group which is divided into low-, average-, and high-ability levels, respectively, with adjustments made for differences in the number of classes spent in completing a given unit of instruction, and $\text{MAc(adj.)}_{\text{CG,LAb}}$, $\text{MAc(adj.)}_{\text{CG,AAb}}$, and $\text{MAc(adj.)}_{\text{CG,HAb}}$ equal the mean achievement scores on a summative posttest for the control group which is divided into low-, average-, and high-ability levels, respectively, with adjustments made for differences in the number of classes spent in completing a given unit of instruction.

As indicated in Table 15, the F ratio for the interaction effects of ability to understand instruction and treatment equaled 3.09; hence, the null hypothesis was rejected at the .05 level of significance. Figure 7 illustrates the graph of the significant ordinal interaction found between ability to understand instruction and quality of instruction relative to achievement scores with adjustments made for differences in the number of classes spent.

For the purpose of ensuring an accurate description of the combination of cells which contributed to the significant ordinal interaction, Table 17 presents the post hoc comparisons of the differences between the differences in cell means. As can be seen from this table, the tetrad difference involving the high- and low-ability levels was significant at the .05 level. The four cells involved in this particular tetrad difference, therefore, combined to effect the significant ordinal interaction between ability to understand instruction and quality of instruction. The tetrad differences involving the high- and average-ability levels, as well as the average- and low-ability levels, were not of a significant nature and, hence, did not encompass cells which contributed to the significant ordinal interaction. The prediction of the experimental consequence which accompanied research hypothesis 5,

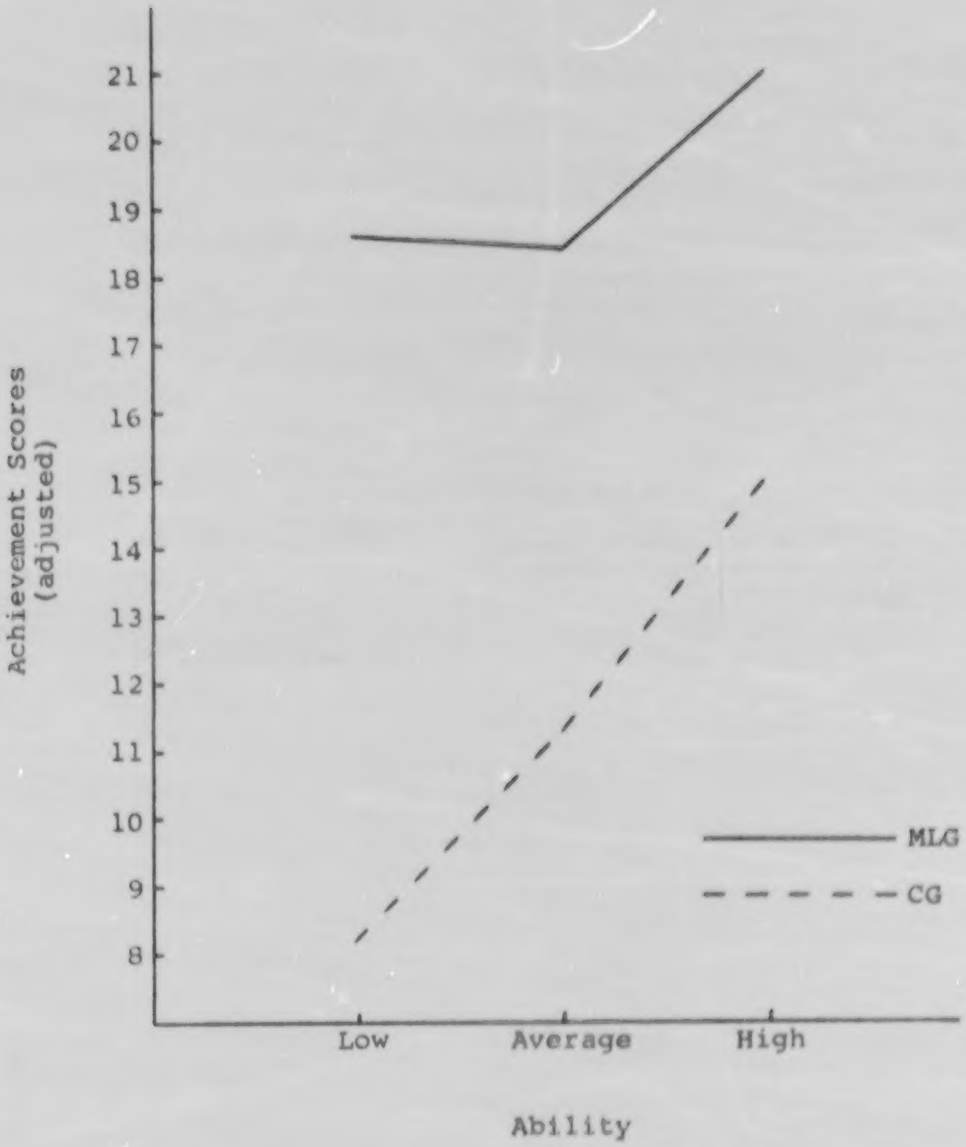

Fig. 7. Illustration of the significant ordinal interaction found between ability to understand instruction and quality of instruction relative to achievement scores with adjustments made for differences in the number of classes spent.

TABLE 17

Post Hoc Comparisons for Significant Ordinal Interaction Discovered Between Ability To Understand Instruction and Quality of Instruction Relative to Achievement Scores (Adjusted for Differences in Number of Classes Spent)

Tetrad Difference[a]	Scheffé Coefficient	Standard Error of Tetrad Difference	Critical Value	Estimate of Tetrad Difference
$\triangle_1 - \triangle_2$	6.00	2.79	±4.09	1.22
$\triangle_1 - \triangle_3$	6.00	2.79	±4.09	4.48*
$\triangle_2 - \triangle_3$	6.00	2.79	±4.09	3.26

[a] $\triangle_1 = \text{MAc(adj.)}_{\text{MLG,HAb}} - \text{MAc(adj.)}_{\text{CG,HAb}} = 5.95$

$\triangle_2 = \text{MAc(adj.)}_{\text{MLG,AAb}} - \text{MAc(adj.)}_{\text{CG,AAb}} = 7.17$

$\triangle_3 = \text{MAc(adj.)}_{\text{MLG,LAb}} - \text{MAc(adj.)}_{\text{CG,LAb}} = 10.43$

*Significant at the .05 level.

though, was accurate in that $MAc(adj.)_{MLG,LAb} - MAc(adj.)_{CG,LAb} > MAc(adj.)_{MLG,AAb} - MAc(adj.)_{CG,AAb} > MAc(adj.)_{MLG,HAb} - MAc(adj.)_{CG,HAb}$.

Hypothesis 6. The sixth question to which an answer was sought in this study asked what the correlation would be between degree of learning and ability to understand instruction in a setting of unlimited opportunity while under a high quality of instruction or a low quality of instruction with the effects due to a rough estimate of perseverance held constant. Research hypothesis 6 predicted that with the effects due to classes spent partialed out, there would be a significant difference between the mastery learning group and the control group relative to the correlation between degree of learning and ability to understand instruction. More specifically, it was predicted that with the effects due to classes spent partialed out, the correlation between degree of learning and ability to understand instruction would not deviate significantly from zero in the mastery learning group but would be significantly positive in the control group, and these correlations would differ significantly from each other. Stated in null form, this hypothesis can be represented as follows:

Ho6: 1. $r_{AcAb.CS,MLG} = 0$

2. $r_{AcAb.CS,CG} = 0$

3. $r_{AcAb.CS,CG} - r_{AcAb.CS,MLG} = 0$

where $r_{AcAb.CS,MLG}$ and $r_{AcAb.CS,CG}$ equal the partial correlations between achievement scores on a summative posttest and ability with adjustments made for differences in the number of classes spent in completing a given unit of instruction for the mastery learning group and the control group, respectively.

Table 18 presents the coefficients of partial correlation between achievement scores and intelligence quotient scores (with the effects due to the number of classes spent partialed out) as well as the standard deviations for both treatment groups. The partial correlation coefficient for the mastery learning group equaled .29 and was significant at the .025 level; hence, the first part of the null hypothesis was rejected. Contrary to the prediction of the first experimental consequence which accompanied research hypothesis 6, it was found that $r_{AcAb.CS,MLG} \nsim 0$.

The partial correlation coefficient for the control group equaled .50 and was significant at the .01 level; hence, the second part of the null hypothesis was rejected. This finding was in accordance with the prediction of the second experimental consequence which accompanied research hypothesis 6 in that $r_{AcAb.CS,CG} > 0$.

Table 19 presents the test for a significant difference between treatment groups relative to the coefficients of partial correlation between achievement scores and intelligence quotient scores with the effects due to the number of classes spent partialed out. Fisher's Z-transformation of the partial correlation coefficients .29 and .50 produced Z_r's of .30 and .55 for the mastery learning group and the control group,

TABLE 18

Coefficients of Partial Correlation Between Achievement Scores and Intelligence Quotient Scores (With Effects Due to Number of Classes Spent Partialed Out) for Mastery Learning and Control Groups

r		Achievement S. D.'s		Intelligence Quotient S. D.'s		Classes Spent S. D.'s		N	
MLG	CG	MLG	CG	MLG	CG	MLG	CG	MLG	CG
.29*	.50**	3.93	5.01	11.35	11.85	5.49	5.21	64	77

*Significant at the .025 level.
**Significant at the .01 level.

TABLE 19

Test for Significant Difference Between Mastery Learning and Control Groups Relative to Coefficients of Partial Correlation Between Achievement Scores and Intelligence Quotient Scores With Effects Due to Number of Classes Spent Partialed Out

r		Z_r				
MLG	CG	MLG	CG	Z_r Diff.	S. E. of Z_r Diff.	$\bar{z}$
.29	.50	.30	.55	.25	.17	1.45

respectively. The resulting $\bar{z}$ equaled 1.45 and was not significant; hence, the third part of the null hypothesis was not rejected. The prediction of the third experimental consequence which accompanied research hypothesis 6 was contradicted in that $r_{AcAb.CS,CG} - r_{AcAb.CS,MLG} \neq 0$ failed to occur at a significant level.

Hypotheses Related to Classes Spent

Preface to hypotheses 7 through 10. Table 20 presents the cell summary for the unweighted means analysis of variance of the number of classes spent by both treatment groups crossed with the three ability levels. The row or ability level means were 12.40, 12.51, and 12.23 for the high-, average-, and low-ability levels, respectively. Table 21 presents the summary table for the unweighted means analysis of variance of the number of classes spent by both treatment groups crossed with three ability levels. The F ratio for the main effects of ability to understand instruction equaled .19 and was not significant. Hence, it was assumed that the failure to obtain significant differences among the three ability levels would serve to preclude the discovery of a significant ordinal interaction between ability to understand instruction and quality of instruction relative to classes spent as predicted in research hypothesis 8.

Hypothesis 7. The seventh question to which an answer was sought in this study asked how quality of instruction would affect a rough estimate of perseverance in a setting of unlimited opportunity. Research hypothesis 7 predicted

TABLE 20

Cell Summary for Unweighted Means Analysis of Variance of Number of Classes Spent by Mastery Learning and Control Groups Crossed With Three Ability Levels

Factor A: Ability	Factor B: Treatments					
	MLG		CG		Rows	
	n	Mean	n	Mean	n	Mean
High	27	16.33	33	9.18	60	12.40
Average	26	16.92	33	9.03	59	12.51
Low	11	15.64	11	8.82	22	12.23
Columns	64	16.45	77	9.06		

TABLE 21

Unweighted Means Analysis of Variance of Number of Classes Spent by Mastery Learning and Control Groups Crossed With Three Ability Levels

Source of Variance	Degrees of Freedom	Sum of Squares	Mean Squares	F
Ability Levels	2	11.21	5.60	.19
Treatments	1	1503.41	1503.41	51.38*
Ability Levels X Treatments	2	5.71	2.86	.10
Error	135	3949.91	29.26	
Total	140	5470.23		

*Significant at the .001 level.

that there would be a significant difference between the mastery learning group and the control group relative to classes spent. More specifically, it was predicted that the mastery learning group would spend a significantly greater number of classes than would the control group. Stated in null form, this hypothesis can be represented as follows:

Ho7: $MCS_{MLG} - MCS_{CG} = 0$

where MCS_{MLG} and MCS_{CG} equal the mean number of classes spent by the mastery learning group and the control group, respectively, in completing a given unit of instruction.

As presented in Table 20, the column or treatment means were 16.45 for the mastery learning group and 9.06 for the control group. According to Table 21, the F ratio for the main effects of treatment equaled 51.38; hence, the null hypothesis was rejected at the .001 level of significance. As evidenced by the aforementioned comparison between the column or treatment means, the prediction of the experimental consequence which accompanied research hypothesis 7 was accurate in that $MCS_{MLG} > MCS_{CG}$.

Hypothesis 8. The eighth question to which an answer was sought in this study asked what the interaction would be between ability to understand instruction and quality of instruction relative to a rough estimate of perseverance in a setting of unlimited opportunity. Research hypothesis 8 predicted that there would be a significant ordinal interaction between ability to understand instruction and quality of instruction relative to classes spent. More specifically, it was predicted that as students decrease in ability to understand instruction, the number of classes spent would increase in the mastery learning group but would decrease in the control group. Stated in null form, this hypothesis can be represented as follows:

$$\text{Ho8:}\quad MCS_{MLG,LAb} - MCS_{CG,LAb} = MCS_{MLG,AAb} - MCS_{CG,AAb} = MCS_{MLG,HAb} - MCS_{CG,HAb}.$$

where $MCS_{MLG,LAb}$, $MCS_{MLG,AAb}$, and $MCS_{MLG,HAb}$ equal the mean number of classes spent in completing a given unit of instruction by the mastery learning group which is divided into low-, average-, and high-ability levels, respectively, and $MCS_{CG,LAb}$, $MCS_{CG,AAb}$, and $MCS_{CG,HAb}$ equal the mean number of classes spent in completing a given unit of instruction by the control group which is divided into low-, average-, and high-ability levels, respectively.

As indicated in Table 21, the F ratio for the interaction effects of ability to understand instruction and treatment equaled .10 and was not significant; hence, the null hypothesis was not rejected. Figure 8 illustrates the graph of the nonsignificant ordinal interaction discovered between ability to understand instruction and quality of instruction relative to the number of classes spent. Though not to a significant extent, the prediction of the experimental consequence which accompanied research hypothesis 8 was contradicted in that $MCS_{MLG,AAb} - MCS_{CG,AAb} > MCS_{MLG,HAb} - MCS_{CG,HAb} > MCS_{MLG,LAb} - MCS_{CG,LAb}$.

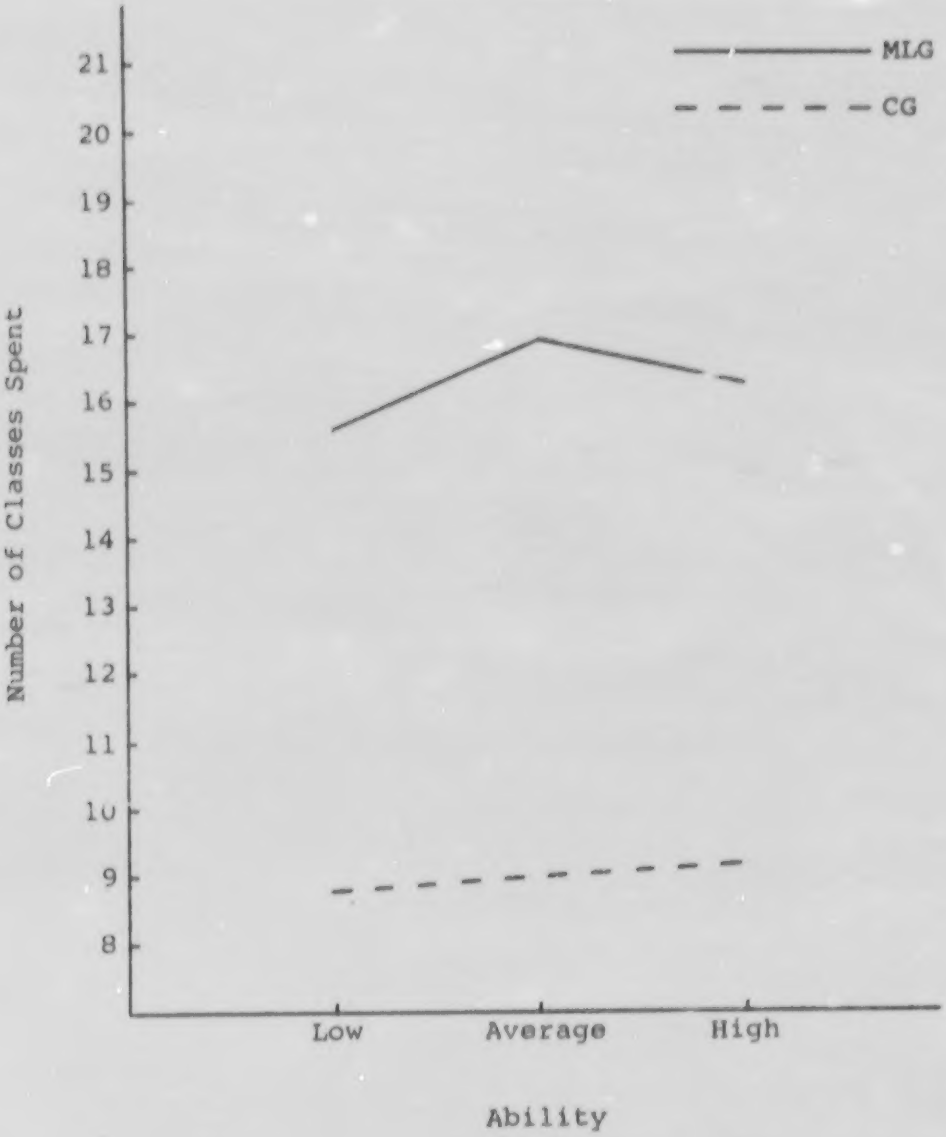

Fig. 8. Illustration of the nonsignificant ordinal interaction found between ability to understand instruction and quality of instruction relative to the number of classes spent.

Hypothesis 9. The ninth question to which an answer was sought in this study asked what the correlation would be between a rough estimate of perseverance and ability to understand instruction in a setting of unlimited opportunity while under a high quality of instruction or a low quality of instruction. Research hypothesis 9 predicted that there would be a significant difference between the mastery learning group and the control group relative to the correlation between classes spent and ability to understand instruction. More specifically, it was predicted that the correlation between classes spent and ability to understand instruction would be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations would differ significantly from each other. Stated in null form, this hypothesis can be represented as follows:

Ho9: 1. $r_{CSAb,MLG} = 0$

2. $r_{CSAb,CG} = 0$

3. $r_{CSAb,CG} - r_{CSAb,MLG} = 0$

where $r_{CSAb,MLG}$ and $r_{CSAb,CG}$ equal the correlations between number of classes spent in completing a given unit of instruction and ability for the mastery learning group and the control group, respectively.

Table 22 presents the coefficients of correlation between the number of classes spent and intelligence quotient scores as well as the standard deviations for both treatment groups. The correlation coefficient for the mastery learning group equaled -.004 and was not significant; hence, the first

TABLE 22

Coefficients of Correlation Between Number of Classes Spent and Intelligence Quotient Scores for Mastery Learning and Control Groups

r		Classes Spent S. D.'s		Intelligence Quotient S. D.'s		N	
MLG	CG	MLG	CG	MLG	CG	MLG	CG
-.004	.04	5.49	5.21	11.35	11.85	64	77

part of the null hypothesis was not rejected. Even though the direction of this correlation was negative, the prediction of the first experimental consequence which accompanied research hypothesis 9 was contradicted in that $r_{CSAb,MLG} < 0$ failed to occur at a significant level.

The correlation coefficient for the control group equaled .04 and was not significant; hence, the second part of the null hypothesis was not rejected. Even though the direction of this correlation was positive, the prediction of the second experimental consequence which accompanied research hypothesis 9 was contradicted in that $r_{CSAb,CG} > 0$ failed to occur at a significant level.

Table 23 presents the test for a significant difference between treatment groups relative to coefficients of correlation between the number of classes spent and intelligence quotient scores. Fisher's Z-transformation of the correlation coefficients -.004 and .04 produced Z_r's of -.004 and .04 for the mastery learning group and the control group, respectively. The resulting $\bar{z}$ equaled .24 and was not significant; hence, the third part of the null hypothesis was not rejected. The prediction of the third experimental consequence which accompanied research hypothesis 9 was contradicted in that $r_{CSAb,CG} - r_{CSAb,MLG} \neq 0$ failed to occur at a significant level.

Hypothesis 10. The tenth question to which an answer was sought in this study asked what the correlation would be between degree of learning and a rough estimate of perseverance

TABLE 23

Test for Significant Difference Between Mastery Learning and Control Groups Relative to Coefficients of Correlation Between Number of Classes Spent and Intelligence Quotient Scores

r		Z_r				
MLG	CG	MLG	CG	Z_r Diff.	S. E. of Z_r Diff.	$\bar{z}$
-.004	.04	-.004	.04	.04	.17	.24

in a setting of unlimited opportunity while under a high quality of instruction or a low quality of instruction. Research hypothesis 10 predicted that there would be a significant difference between the mastery learning group and the control group relative to the correlation between degree of learning and classes spent. More specifically, it was predicted that the correlation between degree of learning and classes spent would be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations would differ significantly from each other. Stated in null form, this hypothesis can be represented as follows:

Ho10: 1. $r_{AcCS,MLG} = 0$

2. $r_{AcCS,CG} = 0$

3. $r_{AcCS,CG} - r_{AcCS,MLG} = 0$

where $r_{AcCS,MLG}$ and $r_{AcCS,CG}$ equal the correlations between achievement scores on a summative posttest and number of classes spent in completing a given unit of instruction for the mastery learning group and the control group, respectively.

Table 24 presents the coefficients of correlation between achievement scores and the number of classes spent as well as the standard deviations for both treatment groups. The correlation coefficient for the mastery learning group equaled -.16 and was not significant; hence, the first part of the null hypothesis was not rejected. Even though the direction of this correlation was negative, the prediction of the first experimental consequence which accompanied

TABLE 24

Coefficients of Correlation Between Achievement Scores and Number of Classes Spent for Mastery Learning and Control Groups

r		Achievement S. D.'s		Classes Spent S. D.'s		N	
MLG	CG	MLG	CG	MLG	CG	MLG	CG
-.16	-.22	3.93	5.01	5.49	5.21	64	77

research hypothesis 10 was contradicted in that $r_{AcCS,MLG} < 0$ failed to occur at a significant level.

The correlation coefficient for the control group equaled -.22 and was not significant; hence, the second part of the null hypothesis was not rejected. The prediction of the second experimental consequence which accompanied research hypothesis 10 was contradicted in that $r_{AcCS,CG} < 0$, though not to a significant extent.

Table 25 presents the test for a significant difference between treatment groups relative to coefficients of correlation between achievement scores and the number of classes spent. Fisher's Z-transformation of the correlation coefficients -.16 and -.22 produced Z_r's of -.16 and -.22 for the mastery learning group and the control group, respectively. The resulting $\bar{z}$ equaled .35 and was not significant; hence, the third part of the null hypothesis was not rejected. The prediction of the third experimental consequence which accompanied research hypothesis 10 was contradicted in that $r_{AcCS,CG} - r_{AcCS,MLG} \neq 0$ failed to occur at a significant level.

Hypotheses Related to Perseverance

Preface to hypotheses 11 through 13. Table 26 presents the cell summary for the unweighted means analysis of variance of the number of minutes spent in persevering by both treatment groups crossed with the three ability levels. The row or ability level means were 8.96, 9.40, and 9.14 for the high-, average-, and low-ability levels, respectively.

TABLE 25

Test for Significant Difference Between Mastery Learning and Control Groups Relative to Coefficients of Correlation Between Achievement Scores and Number of Classes Spent

r		Z_r				
MLG	CG	MLG	CG	Z_r Diff.	S. E. of Z_r Diff.	$\bar{z}$
-.16	-.22	-.16	-.22	.06	.17	.35

TABLE 26

Cell Summary for Unweighted Means Analysis of Variance
of Number of Minutes Spent in Persevering by
Mastery Learning and Control Groups
Crossed With Three Ability Levels

Factor A: Ability	Factor B: Treatments					
	MLG		CG		Rows	
	n	Mean	n	Mean	n	Mean
High	27	8.26	33	9.53	60	8.96
Average	26	10.29	33	8.70	59	9.40
Low	11	10.15	11	8.14	22	9.14
Columns	64	9.41	77	8.97		

Table 27 presents the summary table for the unweighted means analysis of variance of the number of minutes spent in persevering by both treatment groups crossed with the three ability levels. The F ratio for the main effects of ability to understand instruction equaled .20 and was not significant. Hence, it was assumed that the failure to obtain significant differences among the three ability levels would serve to preclude the discovery of a significant ordinal interaction between ability to understand instruction and quality of instruction relative to perseverance as predicted in research hypothesis 12.

Hypothesis 11. The eleventh question to which an answer was sought in this study asked how quality of instruction would affect perseverance in a setting of unlimited opportunity. Research hypothesis 11 predicted that there would be a significant difference between the mastery learning group and the control group relative to perseverance. More specifically, it was predicted that the mastery learning group would manifest a significantly greater amount of perseverance than would the control group. Stated in null form, this hypothesis can be represented as follows:

$$Ho11: \quad MP_{MLG} - MP_{CG} = 0$$

where MP_{MLG} and MP_{CG} equal the mean number of minutes spent by the mastery learning group and the control group, respectively, in persevering on a difficult learning task.

As presented in Table 26, the column or treatment means were 9.41 for the mastery learning group and 8.97 for the

TABLE 27

Unweighted Means Analysis of Variance of Number of Minutes Spent in Persevering by Mastery Learning and Control Groups Crossed With Three Ability Levels

Source of Variance	Degrees of Freedom	Sum of Squares	Mean Squares	F
Ability Levels	2	6.81	3.41	.20
Treatments	1	17.05	17.05	1.00
Ability Levels X Treatments	2	59.81	29.90	1.76
Error	135	2293.36	16.99	
Total	140	2377.03		

control group. According to Table 27, the F ratio for the main effects of treatment equaled 1.00 and was not significant. The prediction of the experimental consequence which accompanied research hypothesis 11 was contradicted in that $MP_{MLG} > MP_{CG}$ failed to occur at a significant level.

Hypothesis 12. The twelfth question to which an answer was sought in this study asked what the interaction would be between ability to understand instruction and quality of instruction relative to perseverance in a setting of unlimited opportunity. Research hypothesis 12 predicted that there would be a significant ordinal interaction between ability to understand instruction and quality of instruction relative to perseverance. More specifically, it was predicted that as students decrease in ability to understand instruction, the amount of perseverance manifested would increase in the mastery learning group but would decrease in the control group. Stated in null form, this hypothesis can be represented as follows:

$$Ho12:\ MP_{MLG,LAb} - MP_{CG,LAb} = MP_{MLG,AAb} - MP_{CG,AAb} = MP_{MLG,HAb} - MP_{CG,HAb}.$$

where $MP_{MLG,LAb}$, $MP_{MLG,AAb}$, and $MP_{MLG,HAb}$ equal the mean number of minutes spent in persevering on a difficult learning task by the mastery learning group which is divided into low-, average-, and high-ability levels, respectively, and $MP_{CG,LAb}$, $MP_{CG,AAb}$, and $MP_{CG,HAb}$ equal the mean number of minutes spent in persevering on a difficult learning task by the control group which is divided into low-, average-, and high-ability levels, respectively.

As indicated in Table 27, the F ratio for the interaction effects of ability to understand instruction and treatment equaled 1.76 and was not significant; hence, the null hypothesis was not rejected. Figure 9 illustrates the graph of the nonsignificant disordinal interaction discovered between ability to understand instruction and quality of instruction relative to the number of minutes spent in persevering. The prediction of the experimental consequence which accompanied research hypothesis 12 was contradicted in that $MP_{MLG,LAb} - MP_{CG,LAb} > MP_{MLG,AAb} - MP_{CG,AAb} > MP_{MLG,HAb} - MP_{CG,HAb}$ failed to occur at a significant level and contributed to a disordinal rather than an ordinal interaction.

Hypothesis 13. The thirteenth question to which an answer was sought in this study asked what the correlation would be between perseverance and ability to understand instruction in a setting of unlimited opportunity while under a high quality of instruction or a low quality of instruction. Research hypothesis 13 predicted that there would be a significant difference between the mastery learning group and the control group relative to the correlation between perseverance and ability to understand instruction. More specifically, it was predicted that the correlation between perseverance and ability to understand instruction would be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations would differ

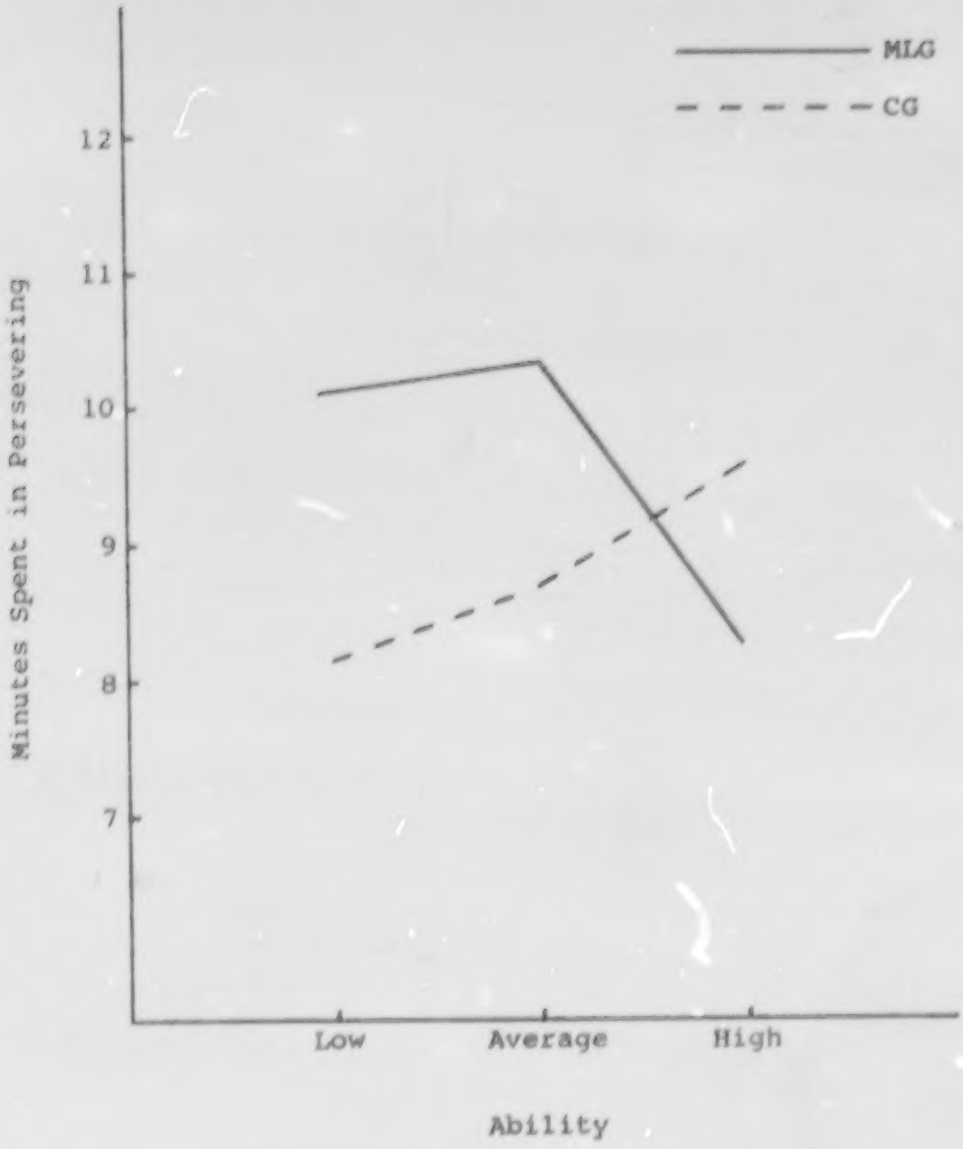

Fig. 9. Illustration of the nonsignificant disordinal interaction found between ability to understand instruction and quality of instruction relative to the number of minutes spent in persevering.

significantly from each other. Stated in null form, this hypothesis can be represented as follows:

Ho13: 1. $r_{PAb,MLG} = 0$

2. $r_{PAb,CG} = 0$

3. $r_{PAb,CG} - r_{PAb,MLG} = 0$

where $r_{PAb,MLG}$ and $r_{PAb,CG}$ equal the correlations between number of minutes spent in persevering on a difficult learning task and ability for the mastery learning group and the control group, respectively.

Table 28 presents the coefficients of correlation between the number of minutes spent in persevering and intelligence quotient scores as well as standard deviations for both treatment groups. The correlation coefficient for the mastery learning group equaled -.19 and was not significant; hence, the first part of the null hypothesis was not rejected. Even though the direction of this correlation was negative, the prediction of the first experimental consequence which accompanied research hypothesis 13 was contradicted in that $r_{PAb,MLG} < 0$ failed to occur at a significant level.

The correlation coefficient for the control group equaled .15 and was not significant; hence, the second part of the null hypothesis was not rejected. Even though the direction of this correlation was positive, the prediction of the second experimental consequence which accompanied research hypothesis 13 was contradicted in that $r_{PAb,CG} > 0$ failed to occur at a significant level.

Table 29 presents the test for a significant difference between treatment groups relative to coefficients of

TABLE 28

Coefficients of Correlation Between Number of Minutes Spent in Persevering and Intelligence Quotient Scores for Mastery Learning and Control Groups

r		Perseverance S. D.'s		Intelligence Quotient S. D.'s		N	
MLG	CG	MLG	CG	MLG	CG	MLG	CG
-.19	.15	4.06	4.19	11.35	11.85	64	77

correlation between the number of minutes spent in persevering and intelligence quotient scores. Fisher's Z-transformation of the correlation coefficients -.19 and .15 produced Z_r's of -.19 and .15 for the mastery learning group and the control group, respectively. The resulting Z equaled 1.97; hence, the third part of the null hypothesis was rejected at the .05 level of significance. This finding was in accordance with the prediction of the third experimental consequence which accompanied research hypothesis 13 in that $r_{PAb,CG} - r_{PAb,MLG} \neq 0$.

Preface to hypotheses 14 through 17. Table 30 presents the cell summary for the unweighted means analysis of covariance of the number of minutes spent in persevering (with achievement scores as the covariate) for both treatment groups crossed with the three ability levels. The adjusted row or ability level means were 8.98, 9.39, and 9.12 for the high-, average-, and low-ability levels, respectively. Table 31 presents the summary table for the unweighted means analysis of covariance of the number of minutes spent in persevering (with achievement as the covariate) for both treatment groups crossed with the three ability levels. The F ratio for the main effects of ability to understand instruction equaled .17 and was not significant. Hence, it was assumed that the failure to obtain significant differences among the three ability levels would serve to preclude the discovery of a significant ordinal interaction between ability to understand instruction and quality of instruction relative

TABLE 29

Test for Significant Difference Between Mastery Learning and Control Groups Relative to Coefficients of Correlation Between Number of Minutes Spent in Persevering and Intelligence Quotient Scores

r		Z_r				
MLG	CG	MLG	CG	Z_r Diff.	S. E. of Z_r Diff.	$\bar{z}$
-.19	.15	-.19	.15	.34	.17	1.97*

*Significant at the .05 level.

TABLE 30

Cell Summary for Unweighted Means Analysis of Covariance of Number of Minutes Spent in Persevering (With Achievement Scores as the Covariate) for Mastery Learning and Control Groups Crossed With Three Ability Levels

Factor A: Ability	Factor B: Treatments											
	MLG				CG				Rows			
		Dependent Variable		Cov.		Dependent Variable		Cov.		Dependent Variable		Cov.
	n	Adj. Mean	Unadj. Mean	Mean	n	Adj. Mean	Unadj. Mean	Mean	n	Adj. Mean	Unadj. Mean	Mean
High	27	8.31	8.26	20.37	33	9.53	9.53	15.64	60	8.98	8.96	17.77
Average	26	10.31	10.29	17.73	33	8.66	8.70	11.91	59	9.39	9.40	14.47
Low	11	10.17	10.15	18.00	11	8.07	8.14	8.73	22	9.12	9.14	13.36
Columns	64	9.44	9.41	18.89	77	8.95	8.97	13.05				

TABLE 31

Unweighted Means Analysis of Covariance of Number of Minutes Spent in Persevering (With Achievement Scores As the Covariate) for Mastery Learning and Control Groups Crossed With Three Ability Levels

Source of Variance	Degrees of Freedom	Sum of Squares	Mean Squares	F
Ability Levels	2	5.96	2.98	.17
Treatments	1	12.95	12.95	.76
Ability Levels X Treatments	2	59.37	29.69	1.73
Error	134	2293.16	17.11	
Total	139	2371.44		

to perseverance (with adjustments made for differences in degree of learning) as predicted in research hypothesis 15.

Hypothesis 14. The fourteenth question to which an answer was sought in this study asked how quality of instruction would affect perseverance in a setting of unlimited opportunity with the effects due to degree of learning held constant. Research hypothesis 14 predicted that with adjustments made for differences in degree of learning, there would be a significant difference between the mastery learning group and the control group relative to perseverance. More specifically, it was predicted that with adjustments made for differences in degree of learning, the mastery learning group would manifest a significantly greater amount of perseverance than would the control group. Stated in null form, this hypothesis can be represented as follows:

$$Ho14: \quad MP(adj.)_{MLG} - MP(adj.)_{CG} = 0$$

where $MP(adj.)_{MLG}$ and $MP(adj.)_{CG}$ equal the mean number of minutes spent by the mastery learning group and the control group, respectively, in persevering on a difficult learning task with adjustments made for differences in achievement scores on the summative posttest.

As presented in Table 30, the adjusted column or treatment means were 9.44 for the mastery learning group and 8.95 for the control group. According to Table 31, the F ratio for the main effects of treatment equaled .76 and was not significant; hence, the null hypothesis was not rejected. The prediction of the experimental consequence which accompanied research hypothesis 14 was contradicted in that $MP(adj.)_{MLG} > MP(adj.)_{CG}$ failed to occur at a significant level.

Hypothesis 15. The fifteenth question to which an answer was sought in this study asked what the interaction would be between ability to understand instruction and quality of instruction relative to perseverance in a setting of unlimited opportunity with the effects due to degree of learning held constant. Research hypothesis 15 predicted that with adjustments made for differences in degree of learning, there would be a significant ordinal interaction between ability to understand instruction and quality of instruction relative to perseverance. More specifically, it was predicted that with adjustments made for differences in degree of learning, as students decrease in ability to understand instruction, the amount of perseverance manifested would increase in the mastery learning group but would decrease in the control group. Stated in null form, this hypothesis can be represented as follows:

$$\begin{aligned} Ho15: \quad & MP(adj.)_{MLG,LAb} - MP(adj.)_{CG,LAb} = \\ & MP(adj.)_{MLG,AAb} - MP(adj.)_{CG,AAb} = \\ & MP(adj.)_{MLG,HAb} - MP(adj.)_{CG,HAb} \end{aligned}$$

where $MP(adj.)_{MLG,LAb}$, $MP(adj.)_{MLG,AAb}$, and $MP(adj.)_{MLG,HAb}$ equal the mean number of minutes spent in persevering on a difficult learning task by the mastery learning group which is divided into low-, average-, and high-ability levels, respectively, with adjustments made for differences in achievement scores on the summative posttest, and $MP(adj.)_{CG,LAb}$, $MP(adj.)_{CG,AAb}$, and $MP(adj.)_{CG,HAb}$ equal the mean number of minutes spent in persevering on a difficult learning task by the control group which is divided into low-, average-, and high-ability levels, respectively, with adjustments made for differences in achievement scores on the summative posttest.

As indicated in Table 31, the F ratio for the interaction effects of ability to understand instruction and treatment equaled 1.73 and was not significant; hence, the null hypothesis was not rejected. Figure 10 illustrates the graph of the nonsignificant disordinal interaction discovered between ability to understand instruction and quality of instruction relative to the number of minutes spent in persevering with adjustments made for differences in achievement scores. The prediction of the experimental consequence which accompanied research hypothesis 15 was contradicted in that $MP(adj.)_{MLG,LAb} - MP(adj.)_{CG,LAb} > MP(adj.)_{MLG,AAb} - MP(adj.)_{CG,AAb} > MP(adj.)_{MLG,HAb} - MP(adj.)_{CG,HAb}$ failed to occur at a significant level and contributed to a disordinal rather than an ordinal interaction.

Hypothesis 16. The sixteenth question to which an answer was sought in this study asked what the correlation would be between perseverance and ability to understand instruction in a setting of unlimited opportunity while under a high quality of instruction or a low quality of instruction with the effects due to degree of learning held constant.

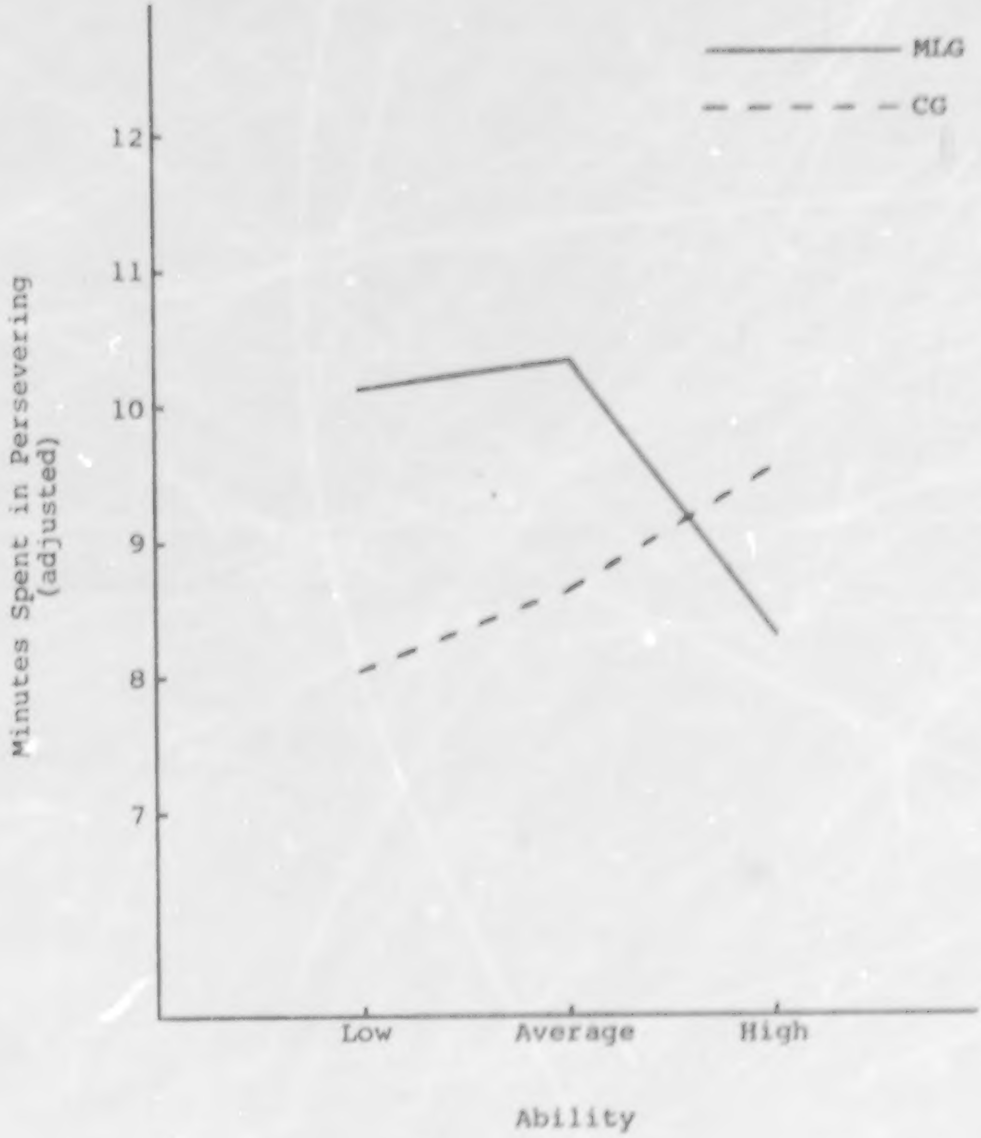

Fig. 10. Illustration of the nonsignificant disordinal interaction found between ability to understand instruction and quality of instruction relative to the number of minutes spent in persevering with adjustments made for differences in achievement scores.

Research hypothesis 16 predicted that with the effects due to degree of learning partialed out, there would be a significant difference between the mastery learning group and the control group relative to the correlation between perseverance and ability to understand instruction. More specifically, it was predicted that with the effects due to degree of learning partialed out, the correlation between perseverance and ability to understand instruction would be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations would differ significantly from each other. Stated in null form, this hypothesis can be represented as follows:

Ho16: 1. $r_{PAb.Ac,MLG} = 0$

2. $r_{PAb.Ac,CG} = 0$

3. $r_{PAb.Ac,CG} - r_{PAb.Ac,MLG} = 0$

where $r_{PAb.Ac,MLG}$ and $r_{PAb.Ac,CG}$ equal the partial correlations between number of minutes spent in persevering on a difficult learning task and ability with adjustments made for differences in achievement scores on the summative posttest for the mastery learning group and the control group, respectively.

Table 32 presents the coefficients of partial correlation between the number of minutes spent in persevering and intelligence quotient scores (with the effects due to achievement scores partialed out) as well as the standard deviations for both treatment groups. The partial correlation

TABLE 32

Coefficients of Partial Correlation Between Number of Minutes Spent in Persevering and Intelligence Quotient Scores (With Effects Due to Achievement Scores Partialed Out) for Mastery Learning and Control Groups

r		Perseverance S. D.'s		Intelligence Quotient S. D.'s		Achievement S. D.'s		N	
MLG	CG	MLG	CG	MLG	CG	MLG	CG	MLG	CG
-.24	.22	4.06	4.19	11.35	11.85	3.93	5.01	64	77

coefficient for the mastery learning group equaled -.24 and was not significant; hence, the first part of the null hypothesis was not rejected. Even though the direction of this partial correlation was negative, the prediction of the first experimental consequence which accompanied research hypothesis 16 was contradicted in that $r_{PAb.Ac,MLG} < 0$ failed to occur at a significant level.

The partial correlation coefficient for the control group equaled .22 and was not significant; hence, the second part of the null hypothesis was not rejected. Even though the direction of this partial correlation was positive, the prediction of the second experimental consequence which accompanied research hypothesis 16 was contradicted in that $r_{PAb.Ac,CG} > 0$ failed to occur at a significant level.

Table 33 presents the test for a significant difference between treatment groups relative to coefficients of partial correlation between the number of minutes spent in persevering and intelligence quotient scores with the effects due to achievement scores partialed out. Fisher's Z-transformation of the partial correlation coefficients -.24 and .22 produced Z_r's of -.25 and .22 for the mastery learning group and the control group, respectively. The resulting Z equaled 2.72; hence, the third part of the null hypothesis was rejected at the .01 level of significance. This finding was in accordance with the prediction of the third experimental consequence which accompanied research hypothesis 16 in that $r_{PAb.Ac,CG} - r_{PAb.Ac,MLG} \neq 0$.

TABLE 33

Test for Significant Difference Between Mastery Learning and Control Groups Relative to Coefficients of Partial Correlation Between Number of Minutes Spent in Persevering and Intelligence Quotient Scores With Effects Due to Achievement Scores Partialed Out

r		Z_r				
MLG	CG	MLG	CG	Z_r Diff.	S. E. of Z_r Diff.	z
-.24	.22	-.25	.22	.47	.17	2.72*

*Significant at the .01 level.

Hypothesis 17. The seventeenth question to which an answer was sought in this study asked what the correlation would be between degree of learning and perseverance in a setting of unlimited opportunity while under a high quality of instruction or a low quality of instruction. Research hypothesis 17 predicted that there would be a significant difference between the mastery learning group and the control group relative to the correlation between degree of learning and perseverance. More specifically, it was predicted that the correlation between degree of learning and perseverance would be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations would differ significantly from each other. Stated in null form, this hypothesis can be represented as follows:

Ho17: 1. $r_{AcP,MLG} = 0$

2. $r_{AcP,CG} = 0$

3. $r_{AcP,CG} - r_{AcP,MLG} = 0$

where $r_{AcP,MLG}$ and $r_{AcP,CG}$ equal the correlations between achievement scores on a summative posttest and number of minutes spent in persevering on a difficult learning task for the mastery learning group and the control group, respectively.

Table 34 presents the coefficients of correlation between achievement scores and the number of minutes spent in persevering as well as the standard deviations for both treatment groups. The correlation coefficient for the mastery

TABLE 34

Coefficients of Correlation Between Achievement Scores and Number of Minutes Spent in Persevering for Mastery Learning and Control Groups

r		Achievement S. D.'s		Perseverance S. D.'s		N	
MLG	CG	MLG	CG	MLG	CG	MLG	CG
.13	-.08	3.93	5.01	4.06	4.19	64	77

learning group equaled .13 and was not significant; hence, the first part of the null hypothesis was not rejected. The prediction of the first experimental consequence which accompanied research hypothesis 17 was contradicted in that $r_{AcP,MLG} > 0$, though not to a significant extent.

The correlation coefficient for the control group equaled -.08 and was not significant; hence, the second part of the null hypothesis was not rejected. The prediction of the second experimental consequence which accompanied research hypothesis 17 was contradicted in that $r_{AcP,CG} < 0$, though not to a significant extent.

Table 35 presents the test for a significant difference between treatment groups relative to coefficients of correlation between achievement scores and the number of minutes spent in persevering. Fisher's Z-transformation of the correlation coefficients .13 and -.08 produced Z_r's of .13 and -.08 for the mastery learning group and the control group, respectively. The resulting Z equaled 1.21 and was not significant; hence, the third part of the null hypothesis was not rejected. The prediction of the third experimental consequence which accompanied research hypothesis 17 was contradicted in that $r_{AcP,CG} - r_{AcP,MLG} \neq 0$ failed to occur at a significant level.

Summary

The 17 null hypotheses that were tested in this study resulted in the following decisions: null hypothesis 1, rejected at the .001 level of significance; null hypothesis 2,

TABLE 35

Test for Significant Difference Between Mastery Learning and Control Groups Relative to Coefficients of Correlation Between Achievement Scores and Number of Minutes Spent in Persevering

r		Z_r		Z_r Diff.	S. E. of Z_r Diff.	$\bar{z}$
MLG	CG	MLG	CG			
.13	-.08	.13	-.08	.21	.17	1.21

rejected at the .05 level of significance; null hypothesis 3 (first part), rejected at the .025 level of significance; null hypothesis 3 (second part), rejected at the .01 level of significance; null hypothesis 3 (third part), failed to reject; null hypothesis 4, rejected at the .001 level of significance; null hypothesis 5, rejected at the .05 level of significance; null hypothesis 6 (first part), rejected at the .025 level of significance; null hypothesis 6 (second part), rejected at the .01 level of significance; null hypothesis 6 (third part), failed to reject; null hypothesis 7, rejected at the .001 level of significance; null hypothesis 8, failed to reject; null hypothesis 9 (first, second, and third parts), failed to reject; null hypothesis 10 (first, second, and third parts), failed to reject; null hypothesis 11, failed to reject; null hypothesis 12, failed to reject; null hypothesis 13 (first and second parts), failed to reject; null hypothesis 13 (third part), rejected at the .05 level of significance; null hypothesis 14, failed to reject; null hypothesis 15, failed to reject; null hypothesis 16 (first and second parts), failed to reject; null hypothesis 16 (third part), rejected at the .01 level of significance; and null hypothesis 17 (first, second, and third parts), failed to reject.

Table 36 presents a convenient summary of the findings as applied to each null hypothesis.

TABLE 36

Summary of the Findings

Null Hypotheses	Results
Ho1: $MAc_{MLG} - MAc_{CG} = 0$	Rejected at the .001 level of significance
Ho2: $MAc_{MLG,LAb} - MAc_{CG,LAb} =$ $MAc_{MLG,AAb} - MAc_{CG,AAb} =$ $MAc_{MLG,HAb} - MAc_{CG,HAb}$	Rejected at the .05 level of significance
Ho3: 1. $r_{AcAb,MLG} = 0$	1. Rejected at the .025 level of significance
2. $r_{AcAb,CG} = 0$	2. Rejected at the .01 level of significance
3. $r_{AcAb,CG} - r_{AcAb,MLG} = 0$	3. Failed to reject
Ho4: $MAc(adj.)_{MLG} - MAc(adj.)_{CG} = 0$	Rejected at the .001 level of significance

Null Hypotheses	Results
Ho5: $MAc(adj.)_{MLG,LAb} - MAc(adj.)_{CG,LAb} =$ $MAc(adj.)_{MLG,AAb} - MAc(adj.)_{CG,AAb} =$ $MAc(adj.)_{MLG,HAb} - MAc(adj.)_{CG,HAb}$	Rejected at the .05 level of significance
Ho6: 1. $r_{AcAb.CS,MLG} = 0$ 2. $r_{AcAb.CS,CG} = 0$ 3. $r_{AcAb.CS,CG} - r_{AcAb.CS,MLG} = 0$	1. Rejected at the .025 level of significance 2. Rejected at the .01 level of significance 3. Failed to reject
Ho7: $MCS_{MLG} - MCS_{CG} = 0$	Rejected at the .001 level of significance
Ho8: $MCS_{MLG,LAb} - MCS_{CG,LAb} =$ $MCS_{MLG,AAb} - MCS_{CG,AAb} =$ $MCS_{MLG,HAb} - MCS_{CG,HAb}$	Failed to reject

	Null Hypotheses	Results
Ho9:	1. $r_{CSAb,MLG} = 0$	1. Failed to reject
	2. $r_{CSAb,CG} = 0$	2. Failed to reject
	3. $r_{CSAb,CG} - r_{CSAb,MLG} = 0$	3. Failed to reject
Ho10:	1. $r_{AcCS,MLG} = 0$	1. Failed to reject
	2. $r_{AcCS,CG} = 0$	2. Failed to reject
	3. $r_{AcCS,CG} - r_{AcCS,MLG} = 0$	3. Failed to reject
Ho11:	$MP_{MLG} - MP_{CG} = 0$	Failed to reject
Ho12:	$MP_{MLG,LAb} - MP_{CG,LAb} = MP_{MLG,AAb} - MP_{CG,AAb} = MP_{MLG,HAb} - MP_{CG,HAb}$	Failed to reject
Ho13:	1. $r_{PAb,MLG} = 0$	1. Failed to reject
	2. $r_{PAb,CG} = 0$	2. Failed to reject
	3. $r_{PAb,CG} - r_{PAb,MLG} = 0$	3. Rejected at the .05 level of significance

Null Hypotheses		Results
Ho14:	$MP(adj.)_{MLG} - MP(adj.)_{CG} = 0$	Failed to reject
Ho15:	$MP(adj.)_{MLG,LAb} - MP(adj.)_{CG,LAb} =$ $MP(adj.)_{MLG,AAb} - MP(adj.)_{CG,AAb} =$ $MP(adj.)_{MLG,HAb} - MP(adj.)_{CG,HAb}$	Failed to reject
Ho16:	1. $r_{PAb.Ac,MLG} = 0$ 2. $r_{PAb.Ac,CG} = 0$ 3. $r_{PAb.Ac,CG} - r_{PAb.Ac,MLG} = 0$	1. Failed to reject 2. Failed to reject 3. Rejected at the .01 level of significance
Ho17:	1. $r_{AcP,MLG} = 0$ 2. $r_{AcP,CG} = 0$ 3. $r_{AcP,CG} - r_{AcP,MLG} = 0$	1. Failed to reject 2. Failed to reject 3. Failed to reject

Discussion of the Findings

Degree of Learning

Main effects of treatment. Research hypothesis 1 predicted that the mastery learning group would attain a significantly greater degree of learning than would the control group. The same assertion was posited by research hypothesis 4 but with the qualification that adjustments would be made for differences in classes spent. In both instances significant results at the .001 level were found in favor of the mastery learning group and, hence, served to confirm that which had been hypothesized.

These results are consistent with the findings of previously cited studies which demonstrated the efficacy of feedback/correction procedures as a high quality of instruction capable of increasing substantially the degree of learning attained by students (Airasian, 1967; Baley, 1972; Block, 1970; Carroll & Spearritt, 1967, Collins, 1969, 1970; Gentile, 1970; Keller, 1968; Kersh, 1970; Kim et al., 1969, 1970; Mayo et al., 1968; Merrill et al., 1970; Moore et al., 1968; Sherman, 1967; Silberman & Coulson, 1964; Thompson, 1941).

Interaction effects of ability to understand instruction and quality of instruction. Research hypothesis 2 predicted that a significant ordinal interaction between ability to understand instruction and quality of instruction relative to degree of learning would be found in accordance with the following pattern: as students decrease in ability to understand instruction, their degree of learning would decrease in both the mastery learning group and the control group; however, the extent of decrease would be significantly greater in the control group than in the mastery learning group. The same assertion was posited by research hypothesis 5 but with the qualification that adjustments would be made for differences in classes spent. In both instances that which had been hypothesized was confirmed by virtue of the following three occurrences: (a) the discovery of a significant interaction at the .05 level; (b) the graphing of the adjusted and unadjusted cell means which indicated the ordinal nature of the interaction; and (c) the identification of a tetrad difference significant at the .05 level and involving the high- and low-ability levels.

These findings are especially noteworthy in that they represent the first instance in which Carroll's hypothesized interaction between ability to understand instruction and quality of instruction has been validated empirically by a statistical test of significance. Although Kim et al. (1969) and Silberman & Coulson (1964) presented data consistent with the pattern identified in Carroll's hypothesized interaction, in each of these studies statistical tests of significance were omitted.

In order to test effectively the nature of Carroll's hypothesized interaction, it was necessary that sharp distinctions among the three ability levels be present. A very important finding in this study concerned the main effects of ability to understand instruction relative to degree of learning which were found to be significant at the .001 level. The application of Scheffé's method of multiple comparisons gave evidence that two of the three possible contrasts involving the three ability levels were significant at the .001 level. Hence, an appropriate situation did exist for a reasonable test of the hypothesized interaction even though one contrast was found to be nonsignificant.

Another finding worthy of mention concerned the absence of a significant tetrad difference involving the high- and average-ability levels as well as the average- and low-ability levels. The various cells so involved apparently failed to contribute to the significant ordinal interaction that resulted. This is perhaps attributable to the ineffectiveness of the treatments in terms of being distinct and powerful enough to affect differentially students of fairly similar rather than just extremely high or low abilities. It must be remembered, though, that Carroll's hypothesized interaction addresses itself to extreme differences in levels of student ability.

Correlation between degree of learning and ability to understand instruction. Research hypothesis 3 predicted that the correlation between degree of learning and ability to understand instruction would not deviate significantly from zero in the mastery learning group but would be significantly positive in the control group, and these correlations would differ significantly from each other. The same assertion was posited by research hypothesis 6 but with the qualification that the effects due to classes spent would be partialed out.

Two of the three findings that resulted were as follows: (a) the correlation and partial correlation coefficients in the mastery learning group were significantly positive at the .025 level and (b) the correlation and partial correlation coefficients in the control group were significantly positive at the .01 level. As evidenced by these first two findings, a significant relationship of a direct nature existed between degree of learning and ability to understand instruction in both treatment groups; however, the extent of this direct relationship was less pronounced in the mastery learning group than in the control group, thus suggesting that a high quality of instruction might be potentially more effective than a low quality of instruction in effecting a lesser dependence of student achievement upon student ability. These results are consistent with data reported by Gaines (1971) and Baley (1972) in which positive correlations between achievement scores and ability scores were in evidence under both high and low qualities of instruction but were less pronounced under the former type of instruction than under the latter. Lewis (1969), however, reported findings in which the correlation coefficients for degree of learning and general intelligence were .80 and .55 under instances of high quality of instruction and low quality of instruction, respectively.

The third finding which resulted was that the correlation and partial correlation coefficients extant in each of the two treatment groups did not differ significantly from each other. Based upon this result, it appears that the relationship between degree of learning and ability to understand instruction might not have been differentially affected by the treatments. Similar results were reported by Gaines (1971). Lewis (1969), however, discovered a significant difference between high and low qualities of instruction relative to the correlation between the achievement and intelligence scores.

Summary. Degree of learning was investigated as a dependent variable in a setting of unlimited opportunity. From this perspective, the results of the study can be summarized as follows:

1. A high quality of instruction characterized by feedback/correction procedures fostered a significantly greater degree of learning among students than did a low quality of instruction characterized by the absence of feedback/correction procedures.

2. The interaction between ability to understand instruction and quality of instruction relative to degree of learning was of a significantly ordinal nature; furthermore, the direction of the interaction was such that students low in ability to understand instruction achieved to a lesser extent when subjected to a low quality of instruction than did students high in ability to understand instruction. This finding was, in effect, a confirmation of the interaction between ability to understand instruction

and quality of instruction as explicitly hypothesized by Carroll relative to degree of learning.

3. The relationship between degree of learning and ability to understand instruction was of a significantly direct nature under both a high quality of instruction and a low quality of instruction. Whether this relationship was less direct under a high quality of instruction than under a low quality of instruction could not be confirmed statistically; however, the data generally supported this view. Concerning this latter point, a feasible explanation might be that the relationship between degree of learning and ability to understand instruction was less direct under a high quality of instruction than under a low quality of instruction due to the fact that the mastery learning group manifested less variance among its achievement scores than did the control group.

Classes Spent

Main effects of treatment. Research hypothesis 7 predicted that the mastery learning group would spend a significantly greater number of classes than would the control group. A significant finding at the .001 level was discovered in favor of the mastery learning group and, hence, served to confirm that which had been hypothesized. Although the mastery learning group, to the exclusion of the control group, was compelled to spend at least two classes in formative trial test sessions, this requirement did not preclude students in the control group from spending as many class sessions as they thought were necessary in order to master the material. Since the mastery learning group spent a significantly greater number of classes than did the control group, it is inferred that the various components of the mastery learning strategy were more effective in ensuring that those students so involved would invest whatever amount of time was necessary in order to attain a high degree of subject-matter mastery.

The finding reported in conjunction with research hypothesis 7, however, is not consistent with the results of an investigation conducted by Merrill et al. (1970). In the aforementioned study the total time spent by students in a mastery learning group on lessons, quizzes, and specific review material was slightly less than the total time spent by members of the control group, even though the former group received more material than the latter. The discrepancy between the finding related to research hypothesis 7 and the results reported by Merrill et al. is perhaps attributable to differing levels of complexity associated with the learning tasks and/or the treatments.

Interaction effects of ability to understand instruction and quality of instruction. Research hypothesis 8 predicted that a significant ordinal interaction between ability to understand instruction and quality of instruction relative to classes spent would be found in accordance with the following pattern: as students decrease in ability to understand instruction, the number of classes spent would increase in the mastery learning group but would decrease in the control group. Although the graph of the cell means indicated an ordinal interaction in which students of low ability to understand instruction spent fewer classes when subjected to a low quality of instruction than did students high in ability to understand instruction, no significant finding was discovered relative to this hypothesis.

This apparent trend, however, is consistent with the pattern of interaction explicitly hypothesized by Carroll relative to degree of learning and logically inferred relative to classes spent. A thorough review of the literature failed to result in the identification of any research evidence that could be compared or contrasted with that which was investigated in research hypothesis 8.

The failure to find the significant ordinal interaction as hypothesized was largely attributable to the nonsignificant result which was obtained for the main effects of ability to understand instruction relative to classes spent. Due to the nature of Carroll's hypothesized interaction, a significant finding with respect to the main effects of ability to understand instruction was a prerequisite to the discovery of the type of significant ordinal interaction predicted by research hypothesis 8.

Correlation between ability to understand instruction and quality of instruction. Research hypothesis 9 predicted that the correlation between classes spent and ability to understand instruction would be significantly negative in the mastery learning group but significantly positive in the control group and that these correlations would differ significantly from each other. The three findings which resulted were as follows: (a) the correlation coefficient in the mastery learning group was negative, though not to a significant extent; (b) the correlation coefficient in the control group was positive, though not to a significant extent; and (c) the correlation coefficients extant in each of the two treatment groups did not differ significantly from each other.

Although none of the findings occurred at a significant level, the relationship between classes spent and ability to understand instruction was of an inverse nature in the mastery learning group but of a direct nature in the control group. Though not confirmed statistically, this is possibly suggestive of the potential effectiveness of a high quality of instruction in encouraging students of low ability to spend a greater amount of time attempting to attain mastery than would students of comparable ability who are exposed to a low quality of instruction. With respect to the third finding, it appears that the relationship between classes spent and ability to understand instruction might not have been differentially affected by the treatments.

Correlation between degree of learning and classes spent. Research hypothesis 10 predicted that the correlation between degree of learning and classes spent would be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations would differ significantly from each other. The three findings which resulted were as follows: (a) the correlation coefficient in the mastery learning group was negative, though not to a significant extent; (b) the correlation coefficient in the control group, likewise, was negative, though not to a significant extent; and (c) the correlation coefficients extant in each of the two treatment groups did not differ significantly from each other.

Although none of the findings occurred at a significant level, the relationship between degree of learning and classes spent was of an inverse nature in both the mastery learning group and the control group. The first finding reported above is consistent with data reported by Airasian (1967) in which the correlation between achievement scores and total hours spent in weekly study by students in a mastery learning group resulted in coefficients which ranged from .07 to -.46 over a ten-week period. With respect to the third finding, it appears that the relationship between degree of learning and classes spent might not have been differentially affected by the treatments.

Summary. The dependent variable labeled classes spent was investigated as a rough estimate of perseverance in a setting of unlimited opportunity. From this perspective, the results of the study can be summarized as follows:

1. A high quality of instruction characterized by feedback/correction procedures fostered a significantly

greater number of classes spent by students than did a low quality of instruction characterized by the absence of feedback/correction procedures.

2. The interaction between ability to understand instruction and quality of instruction relative to classes spent was of a nonsignificantly ordinal nature; furthermore, the direction of the interaction was such that students low in ability to understand instruction spent a fewer number of classes when subjected to a low quality of instruction than did students high in ability to understand instruction. The direction of the interaction was consistent with the interaction explicitly hypothesized by Carroll relative to degree of learning and logically inferred relative to classes spent.

3. Though not confirmed statistically, the relationship between classes spent and ability to understand instruction was of an inverse nature under a high quality of instruction but of a direct nature under a low quality of instruction; furthermore, it appeared that the relationship between classes spent and ability to understand instruction was not differentially affected by the treatments.

4. Though not confirmed statistically, the relationship between degree of learning and classes spent was of an inverse nature under both a high quality of instruction and a low quality of instruction. Whether this relationship was less of an inverse nature under a high quality of instruction than under a low quality of instruction could not be confirmed statistically; however, the data generally supported this view.

Concerning this latter point, a feasible explanation might be that the relationship between degree of learning and classes spent was less of an inverse nature under a high quality of instruction than under a low quality of instruction due to the fact that the mastery learning group manifested less variance among its achievement scores than did the control group.

Perseverance

Main effects of treatment. Research hypothesis 11 predicted that the mastery learning group would manifest a significantly greater amount of perseverance than would the control group. The same assertion was posited by research hypothesis 14 but with the qualification that adjustments would be made for differences in degree of learning. Although in both instances the mastery learning group persevered for a longer period of time than did the control group, no significant differences resulted. It appears, therefore, that the perseverance of students on a difficult learning task might not have been differentially affected by the treatments to the extent hypothesized. These findings are consistent with the results reported by Carroll & Spearritt (1967) in which perseverance on a difficult post-experimental task was greater in a mastery learning group than in a control group, though not to a significant extent.

Interaction effects of ability to understand instruction and quality of instruction. Research hypothesis 12 predicted that a significant ordinal interaction between ability to understand instruction and quality of instruction relative to perseverance would be found in accordance with the following pattern: as students decrease in ability to understand instruction, the amount of perseverance manifested would increase in the mastery learning group but would decrease in the control group. The same assertion was posited by research hypothesis 15 but with the qualification that adjustments would be made for differences in degree of learning. In both instances a nonsignificant disordinal interaction was discovered in which students of low ability to understand instruction manifested a lesser amount of perseverance when subjected to a low quality of instruction than did students high in ability to understand instruction. The direction of this disordinal interaction was such that the difference between treatment groups relative to perseverance increased as students decreased in ability to understand instruction; furthermore, only at the high-ability level did the control group manifest a greater amount of perseverance than did the mastery learning group.

The disordinal (though nonsignificant) interaction reported here, but not the direction of it, is similar to the findings of a previously cited study by Carroll & Spearritt (1967) in which a significant disordinal interaction between ability to understand instruction and quality of instruction relative to perseverance was discovered. This present study differs somewhat from the investigation reported by Carroll & Spearritt in that the former does not give evidence of a pattern of interaction which would contradict a logical extension of Carroll's hypothesized interaction to the effect that students of low ability persevere less while under a low quality of instruction than do students of high ability. As discussed in Chapter 2, the study by Carroll & Spearritt gave evidence that students exposed to a low quality of instruction were less willing to persevere on a difficult post-experimental task if they were in the high- or low- but not middle-ability level.

It is assumed that the nonsignificant interaction which resulted is largely attributable to the absence of a significant main effects of ability to understand instruction relative to perseverance. Due to the nature of Carroll's hypothesized interaction, a significant finding with respect to the main effects of ability to understand instruction was a prerequisite to the discovery of the type of significant ordinal interaction predicted by research hypotheses 12 and 15.

Correlation between perseverance and ability to understand instruction. Research hypothesis 13 predicted that the correlation between perseverance and ability to understand instruction would be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations would differ significantly from each other. The same assertion was posited by research hypothesis 16 but with the qualification that the effects due to degree of learning would be partialed out. In both instances the following three findings were reported:

(a) the correlation and partial correlation coefficients in the mastery learning group were negative, though not to a significant extent; (b) the correlation and partial correlation coefficients in the control group were positive, though not to a significant extent; and (c) the correlation coefficients extant in each of the two treatment groups differed significantly from each other at the .05 level while the partial correlation coefficients extant in each of the two treatment groups differed significantly from each other at the .01 level.

Although the first two findings were not reported at a significant level, the relationship between perseverance and ability to understand instruction was of an inverse nature in the mastery learning group but of a direct nature in the control group. Though not confirmed statistically, this is possibly suggestive of the potential effectiveness of a high quality of instruction in encouraging students of low ability to manifest a greater amount of perseverance on a difficult learning task than would students of comparable intelligence who are exposed to a low quality of instruction. With respect to the third group of findings, the relationship between perseverance and ability to understand instruction was differentially affected by the treatments.

Correlation between degree of learning and perseverance. Research hypothesis 17 predicted that the correlation between degree of learning and perseverance would be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations would differ significantly from each other. The three findings which resulted were as follows: (a) the correlation coefficient in the mastery learning group was positive, though not to a significant extent; (b) the correlation coefficient in the control group was negative, though not to a significant extent; and (c) the correlation coefficients extant in each of the two treatment groups did not differ significantly from each other.

Although none of the findings occurred at a significant level, the relationship between degree of learning and perseverance was of a direct nature in the mastery learning group but of an inverse nature in the control group. With respect to the third finding, it appears that the relationship between degree of learning and perseverance might not have been differentially affected by the treatments.

Summary. Perseverance was investigated as a dependent variable in a setting of unlimited opportunity. As alluded to earlier, a limitation of this study relative to the measurement of perseverance encompassed the fact that students who were tested individually were not subjected to the peer-group influence which was a part of those situations in which a group of two to five students were involved. From the perspective of perseverance as a dependent variable, the results of the study can be summarized as follows:

1. A high quality of instruction characterized by feedback/correction procedures did not foster a significantly

greater amount of perseverance among students than did a low quality of instruction characterized by the absence of feedback/correction procedures.

2. The interaction between ability to understand instruction and quality of instruction relative to perseverance was of a nonsignificantly disordinal nature; furthermore, the direction of the interaction was such that students low in ability to understand instruction persevered less when subjected to a low quality of instruction than did students high in ability to understand instruction. The direction of the interaction was consistent with the interaction explicitly hypothesized by Carroll relative to degree of learning and logically inferred relative to perseverance.

3. Though not confirmed statistically, the relationship between perseverance and ability to understand instruction was of an inverse nature under a high quality of instruction but of a direct nature under a low quality of instruction. To a significant extent, though, the relationship between perseverance and ability to understand instruction was differentially affected by the treatments.

4. Though not confirmed statistically, the relationship between degree of learning and perseverance was of a direct nature under a high quality of instruction but of an inverse nature under a low quality of instruction. Whether these two opposite types of relationships differed substantially from each other could not be confirmed statistically; however, the data generally supported this view. Concerning

this latter point, a feasible explanation might be that the direct relationship between degree of learning and perseverance under a high quality of instruction and the inverse relationship between degree of learning and perseverance under a low quality of instruction appeared to differ substantially from each other due to the fact that the mastery learning group manifested less variance among its achievement scores than did the control group.

CHAPTER 5

Summary, Conclusions, Implications, and Recommendations

Summary

The Carroll Model of School Learning

John B. Carroll's (1963) model of school learning is a paradigm which describes the degree of learning that takes place in a school setting as a function of the time spent on a learning task divided by the time needed for its mastery. The basic formulation of the model can be expressed as follows:

$$\text{Degree of Learning} = f\,\frac{(\text{Time Spent})}{(\text{Time Needed})}$$

The following five variables comprise the model: (a) opportunity--the amount of time allowed or made available for learning; (b) perseverance--the amount of time the learner is willing to spend actively engaged in a learning task; (c) aptitude--the amount of time the student will need to learn the task under optimal instructional conditions; (d) ability to understand instruction--the ability of the learner to understand the nature of the task he is to learn and the procedures he is to follow in the learning of the task, a combination of general and verbal intelligence; (e) quality of instruction--the degree to which the presentation, explanation, and ordering of elements of the task to be learned approach the optimum for a given learner. Opportunity and perseverance function as determinants of time spent while aptitude, ability to understand instruction, and quality of instruction serve as determinants of time needed.

Purpose

The theoretical rationale frequently identified as the basis underlying the nongraded organizational structuring of schools has been the position that individual learners differ with respect to their potentialities for achievement and interest in various subject areas and, therefore, must be permitted to operate under a form of school organization which is amenable--and indeed conducive--to each student progressing at a rate dictated by his own capabilities. Due to this theoretical orientation, past research concerning the vertical structuring of schools has attempted to demonstrate the superiority of nongradedness over gradedness as the more viable organizational approach for accommodating the individual differences among students. Unfortunately, the literature currently available on alternative forms of school organization can be characterized as barren relative to the identification and investigation of a specific model which might serve as a theoretical justification for the manner in which schools are organized. This study, however, asserted that Carroll's (1963) model of school learning does represent at least a potentially tenable basis for decisions in this area of administration.

Predicated upon the aforementioned facts, the objectives of this investigation encompassed the following:

1. To utilize the Carroll model as a framework for implementing a mastery learning strategy in a nongraded setting.

2. To identify the Carroll model as a possible theoretical basis for administrative decisions regarding the organizational structuring of schools.

3. To test certain hypotheses derived from the model which have implications concerning school organization.

Research Questions

The implementation of this study sought to provide answers to the following questions:

1. How will quality of instruction affect degree of learning in a setting of unlimited opportunity?

2. What will be the interaction between ability to understand instruction and quality of instruction relative to degree of learning in a setting of unlimited opportunity?

3. What will be the correlation between degree of learning and ability to understand instruction in a setting of unlimited opportunity while under a high quality of instruction? while under a low quality of instruction?

4. With the effects due to a rough estimate of perseverance held constant, how will quality of instruction affect degree of learning in a setting of unlimited opportunity?

5. With the effects due to a rough estimate of perseverance held constant, what will be the interaction between

ability to understand instruction and quality of instruction relative to degree of learning in a setting of unlimited opportunity?

6. With the effects due to a rough estimate of perseverance held constant, what will be the correlation between degree of learning and ability to understand instruction in a setting of unlimited opportunity while under a high quality of instruction? while under a low quality of instruction?

7. How will quality of instruction affect a rough estimate of perseverance in a setting of unlimited opportunity?

8. What will be the interaction between ability to understand instruction and quality of instruction relative to a rough estimate of perseverance in a setting of unlimited opportunity?

9. What will be the correlation between a rough estimate of perseverance and ability to understand instruction in a setting of unlimited opportunity while under a high quality of instruction? while under a low quality of instruction?

10. What will be the correlation between degree of learning and a rough estimate of perseverance in a setting of unlimited opportunity while under a high quality of instruction? while under a low quality of instruction?

11. How will quality of instruction affect perseverance in a setting of unlimited opportunity?

12. What will be the interaction between ability to understand instruction and quality of instruction relative to perseverance in a setting of unlimited opportunity?

13. What will be the correlation between perseverance and ability to understand instruction in a setting of unlimited opportunity while under a high quality of instruction? while under a low quality of instruction?

14. With the effects due to degree of learning held constant, how will quality of instruction affect perseverance in a setting of unlimited opportunity?

15. With the effects due to degree of learning held constant, what will be the interaction between ability to understand instruction and quality of instruction relative to perseverance in a setting of unlimited opportunity?

16. With the effects due to degree of learning held constant, what will be the correlation between perseverance and ability to understand instruction in a setting of unlimited opportunity while under a high quality of instruction? while under a low quality of instruction?

17. What will be the correlation between degree of learning and perseverance in a setting of unlimited opportunity while under a high quality of instruction? while under a low quality of instruction?

Research Hypotheses

The following research hypotheses were identified as being of significant relevance to this study and, hence, were investigated:

Research hypothesis 1. There will be a significant difference between the mastery learning group and the control group relative to degree of learning. More specifically, the mastery learning group will attain a significantly greater degree of learning than will the control group.

Research hypothesis 2. There will be a significant ordinal interaction between ability to understand instruction and quality of instruction relative to degree of learning. More specifically, as students decrease in ability to understand instruction, their degree of learning will decrease in both the mastery learning group and the control group; however, the extent of decrease will be significantly greater in the control group than in the mastery learning group.

Research hypothesis 3. There will be a significant difference between the mastery learning group and the control group relative to the correlation between degree of learning and ability to understand instruction. More specifically, the correlation between degree of learning and ability to understand instruction will not deviate significantly from zero in the mastery learning group but will be significantly positive in the control group, and these correlations will differ significantly from each other.

Research hypothesis 4. With adjustments made for differences in classes spent, there will be a significant difference between the mastery learning group and the control group relative to degree of learning. More specifically, with adjustments made for differences in classes spent, the mastery learning group will attain a significantly greater degree of learning than will the control group.

Research hypothesis 5. With adjustments made for differences in classes spent, there will be a significant ordinal interaction between ability to understand instruction and quality of instruction relative to degree of learning. More specifically, with adjustments made for differences in classes spent, as students decrease in ability to understand instruction, their degree of learning will decrease in both the mastery learning group and the control group; however, the extent of decrease will be significantly greater in the control group than in the mastery learning group.

Research hypothesis 6. With the effects due to classes spent partialed out, there will be a significant difference between the mastery learning group and the control group relative to the correlation between degree of learning and ability to understand instruction. More specifically, with the effects due to classes spent partialed out, the correlation between degree of learning and ability to understand instruction will not deviate significantly from zero in the mastery learning group but will be significantly positive in the control group, and these correlations will differ significantly from each other.

Research hypothesis 7. There will be a significant difference between the mastery learning group and the control group relative to classes spent. More specifically, the mastery learning group will spend a significantly greater number of classes than will the control group.

Research hypothesis 8. There will be a significant ordinal interaction between ability to understand instruction and quality of instruction relative to classes spent. More specifically, as students decrease in ability to understand instruction, the number of classes spent will increase in the mastery learning group but will decrease in the control group.

Research hypothesis 9. There will be a significant difference between the mastery learning group and the control group relative to the correlation between classes spent and ability to understand instruction. More specifically, the correlation between classes spent and ability to understand instruction will be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations will differ significantly from each other.

Research hypothesis 10. There will be a significant difference between the mastery learning group and the control group relative to the correlation between degree of learning and classes spent. More specifically, the correlation between degree of learning and classes spent will be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations will differ significantly from each other.

Research hypothesis 11. There will be a significant difference between the mastery learning group and the control group relative to perseverance. More specifically, the mastery learning group will manifest a significantly greater amount of perseverance than will the control group.

Research hypothesis 12. There will be a significant ordinal interaction between ability to understand instruction and quality of instruction relative to perseverance. More specifically, as students decrease in ability to understand instruction, the amount of perseverance manifested will increase in the mastery learning group but will decrease in the control group.

Research hypothesis 13. There will be a significant difference between the mastery learning group and the control group relative to the correlation between perseverance and ability to understand instruction. More specifically, the correlation between perseverance and ability to understand instruction will be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations will differ significantly from each other.

Research hypothesis 14. With adjustments made for differences in degree of learning, there will be a significant difference between the mastery learning group and the control group relative to perseverance. More specifically, with adjustments made for differences in degree of learning, the mastery learning group will manifest a significantly greater amount of perseverance than will the control group.

Research hypothesis 15. With adjustments made for differences in degree of learning, there will be a significant ordinal interaction between ability to understand instruction and quality of instruction relative to perseverance. More specifically, with adjustments made for differences in degree of learning, as students decrease in ability to understand instruction, the amount of perseverance manifested will increase in the mastery learning group but will decrease in the control group.

Research hypothesis 16. With the effects due to differences in degree of learning partialed out, there will be a significant difference between the mastery learning group and the control group relative to the correlation between perseverance and ability to understand instruction. More specifically, with the effects due to degree of learning partialed out, the correlation between perseverance and ability to understand instruction will be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations will differ significantly from each other.

Research hypothesis 17. There will be a significant difference between the mastery learning group and the control group relative to the correlation between degree of learning and perseverance. More specifically, the correlation between degree of learning and perseverance will be significantly negative in the mastery learning group but significantly positive in the control group, and these correlations will differ significantly from each other.

Research Sample

The sample used in this investigation was identical to a population of 169 male students who were enrolled in the second of six learning sequences which comprised an Algebra I course at the secondary school level. Intelligence quotient scores were used for dividing this sample into three levels of ability. The students within each of the three ability levels were randomly assigned to two levels of treatment designated as the mastery learning group and the control group. Initially, there were 85 and 84 students in the mastery learning group and the control group, respectively. As a result of experimental mortality, the size of the mastery learning group and the control group was reduced to 64 and 77 students, respectively; hence, the final sample size equaled 141 students.

Treatments

The treatment to which the mastery learning group was exposed included (a) the specification of performance objectives, (b) the use of formative trial tests, and (c) the prescription of learning correctives of a review and/or remedial nature. From the perspective of Carroll's model, this instructional strategy was viewed as comprising a high quality of instruction.

The treatment to which the control group was exposed included the specification of performance objectives but did not encompass any procedures of a feedback/correction nature.

From the perspective of Carroll's model, this pedagogical approach was viewed as comprising a low quality of instruction.

Data Collection Procedures

Degree of learning, number of classes spent, and perseverance on a difficult learning task were the three dependent variables investigated in this study. Degree of learning was assessed by way of achievement scores attained on a summative posttest based upon the specific learning tasks to which the students were exposed.

Concerning the tabulation of the number of classes spent, an accurate account was maintained of the total number of instructional periods in algebra attended by each student while completing the designated unit of instruction. This record was amassed by way of a simple attendance check. Excluded from this tabulation were the two class sessions devoted by each student to the completion of the summative posttest and the Assessment of Perseverance. Included in this tabulation, though, were the number of class sessions spent by each member of the mastery learning group in completing the two formative trial tests.

The instrument labeled Assessment of Perseverance was used to measure the amount of time a student would be willing to spend actively engaged with a difficult learning task. Subsequent to the completion of the summative posttest, each student was given the Assessment of Perseverance instrument which consisted of (a) explanatory material on a new algebraic topic and (b) a single mathematical problem pertaining to the same topic. In a highly controlled setting, each student was requested to read the instructional material and then to solve the problem. Measures of perseverance were then obtained by way of the total number of minutes and seconds spent by each subject on the learning task.

Research Design and Data Analysis Procedures

The experimental design used in this study can be characterized as a logical extension and concurrent replication of the Posttest-Only Control Group Design as presented by Campbell & Stanley (1963). The crossing of three ability levels with two treatment levels resulted in a 3 X 2 fixed-effects factorial design.

The various statistical techniques that were employed included the following: two-way fixed-effects analysis of variance (unweighted means), two-way fixed-effects analysis of covariance (unweighted means), Pearson product-moment correlation, partial correlation, Fisher's Z-transformation of r, Scheffe's test for multiple comparisons, and Marascuilo & Levin's (1970) post hoc comparisons for interaction.

Limitations

The various limitations inherent in this study included the following:

1. The reactive effects of experimental procedures upon students represented a threat to the external validity of the study.

2. The relatively long duration of the investigation--September 24, 1973 to December 7, 1973--represented a threat to the internal validity of the study known as contemporary history.

3. Experimental mortality accounted for a total of 28 students who were lost to the study and, hence, threatened the internal validity of the investigation. Twenty-one of the twenty-eight students who were eliminated had been members of the mastery learning group.

4. The dependent variable designated as classes spent was admittedly only a rough approximation of the variable identified by Carroll as perseverance.

5. A measurements limitation of this study relative to perseverance as a dependent variable encompassed the fact that students who were administered the Assessment of Perseverance individually were not subjected to the peer-group influence which was a part of those situations in which a group of two to five students were tested simultaneously.

Findings

The findings reported in this study relative to each of the seventeen null hypotheses are summarized in this section. In order to facilitate the synthesis of these findings, the various null hypotheses are grouped according to pertinent dependent variables and statistical techniques used:

Degree of learning.

Ho1 and Ho4: The null hypothesis of no significant difference between the mastery learning group and the control group relative to degree of learning, inclusive or exclusive of the effects due to classes spent, was rejected at the .001 level of significance in favor of the mastery learning group.

Ho2 and Ho5: The null hypothesis of no significant interaction of ability to understand instruction and treatment relative to degree of learning, inclusive or exclusive of the effects due to classes spent, was rejected at the .05 level of significance in favor of an ordinal interaction.

Ho3: The null hypothesis of no significant difference between (a) the mastery learning group's correlation coefficient for degree of learning and ability to understand instruction and (b) zero was rejected at the .025 level of significance in favor of a positive coefficient. The null hypothesis of no significant difference between (a) the control group's correlation coefficient for degree of learning and ability to understand instruction and (b) zero was rejected at the .01 level of significance in favor of a positive coefficient. The null hypothesis of no significant difference between the mastery learning and the control groups' correlation coefficients for degree of learning and ability to understand instruction was not rejected.

Ho6: The null hypothesis of no significant difference between (a) the mastery learning group's partial correlation coefficient for degree of learning and ability to understand with the effects due to classes spent partialed out and (b) zero was rejected at the .025 level of significance in favor of a positive coefficient. The null hypothesis of no significant difference between (a) the control group's partial correlation coefficient for degree of learning and ability to understand instruction with the effects due to classes spent partialed out and (b) zero was rejected at the .01 level of significance in favor of a positive coefficient. The null hypothesis of no significant difference between the mastery learning and the control groups' partial correlation coefficients for degree of learning and ability to understand instruction with the effects due to classes spent partialed out was not rejected.

Classes spent.

Ho7: The null hypothesis of no significant difference between the mastery learning group and the control group relative to classes spent was rejected at the .001 level of significance in favor of the mastery learning group.

Ho8: The null hypothesis of no significant interaction of ability to understand instruction and treatment relative to classes spent was not rejected.

Ho9: The null hypothesis of no significant difference between (a) the mastery learning group's correlation coefficient for classes spent and ability to understand instruction and (b) zero was not rejected. The null hypothesis of no significant difference between (a) the control group's correlation coefficient for classes spent and ability to understand instruction and (b) zero was not rejected. The null hypothesis of no significant difference between the mastery learning and the control groups' correlation coefficients for classes spent and ability to understand instruction was not rejected.

Ho10: The null hypothesis of no significant difference between (a) the mastery learning group's correlation coefficient for degree of learning and classes spent and (b) zero was not rejected. The null hypothesis of no significant difference between (a) the control group's correlation coefficient for degree of learning and classes spent and (b) zero was not rejected. The null hypothesis of no significant difference between the mastery learning and the control groups' correlation coefficients for degree of learning and classes spent was not rejected.

Perseverance.

Ho11 and Ho14: The null hypothesis of no significant difference between the mastery learning group and the control group relative to perseverance, inclusive or exclusive of the effects due to degree of learning, was not rejected.

Ho12 and Ho15: The null hypothesis of no significant interaction of ability to understand instruction and treatment relative to perseverance, inclusive or exclusive of the effects due to degree of learning, was not rejected.

Ho13: The null hypothesis of no significant difference between (a) the mastery learning group's correlation coefficient for perseverance and ability to understand instruction and (b) zero was not rejected. The null hypothesis of no significant difference between (a) the control group's correlation coefficient for perseverance and ability to understand instruction and (b) zero was not rejected. The null hypothesis of no significant difference between the mastery learning and the control groups' correlation coefficients for perseverance and ability to understand instruction was rejected at the .05 level of significance.

Ho16: The null hypothesis of no significant difference between (a) the mastery learning group's partial correlation coefficient for perseverance and ability to understand instruction with the effects due to degree of learning partialed out and (b) zero was not rejected. The null hypothesis of no significant difference between (a) the control group's partial correlation coefficient for perseverance and ability to understand instruction with the effects due to degree of learning partialed out and (b) zero was not rejected. The null hypothesis of no significant difference between the mastery learning and the control groups' partial correlation coefficients for perseverance and ability to understand instruction with the effects due to degree of learning partialed out was rejected at the .01 level of significance.

Ho17: The null hypothesis of no significant difference between (a) the mastery learning group's correlation coefficient for degree of learning and perseverance and (b) zero was not rejected. The null hypothesis of no significant difference between (a) the control group's correlation coefficient for degree of learning and perseverance and (b) zero was not rejected. The null hypothesis of no significant difference between the mastery learning and the control groups' correlation coefficients for degree of learning and perseverance was not rejected.

Conclusions

Based upon a careful analysis of the findings presented in this study, it would seem that the following conclusions are warranted:

1. A high quality of instruction characterized by feedback/correction procedures fostered a significantly greater degree of learning among students than did a low quality of instruction characterized by the absence of feedback/correction procedures. This conclusion serves to confirm statistically the following assumption inherent within the Carroll model: As quality of instruction increases while the other components remain constant, there is a corresponding decrease in the time needed by a student for learning, thus resulting in a closer approximation of mastery learning.

2. The interaction between ability to understand instruction and quality of instruction relative to degree of learning was of a significantly ordinal nature; furthermore, the direction of the interaction was such that students low in ability to understand instruction achieved to a lesser extent when subjected to a low quality of instruction than did students high in ability to understand instruction. This conclusion serves to confirm statistically the interaction between ability to understand instruction and quality

of instruction as explicitly hypothesized by Carroll relative to degree of learning as well as the following logical argument which undergirds it: (a) Low ability to understand instruction implies a lesser degree of learning than does high ability to understand instruction. (b) Low quality of instruction implies a lesser degree of learning than does high quality of instruction. (c) Therefore, as explicitly hypothesized by Carroll, the interaction between ability to understand instruction and quality of instruction relative to degree of learning is such that students low in ability to understand instruction achieve less when subjected to low quality of instruction than do students high in ability to understand instruction.

3. The relationship between degree of learning and ability to understand instruction was of a significantly direct nature under both a high quality of instruction and a low quality of instruction. Whether this relationship was less direct under a high quality of instruction than under a low quality of instruction could not be confirmed statistically although the data generally supported this view. Hence, the following logical argument was not confirmed statistically: (a) Low ability to understand instruction implies a lesser degree of learning than does high ability to understand instruction. (b) High quality of instruction implies a greater degree of learning than does low quality of instruction. (c) Therefore, degree of learning is less a direct function of ability to understand instruction under

high quality of instruction than under low quality of instruction.

4. A high quality of instruction characterized by feedback/correction procedures fostered a significantly greater amount of perseverance (roughly estimated) among students than did a low quality of instruction characterized by the absence of feedback/correction procedures. Hence, the following logical argument was confirmed statistically: (*a*) High quality of instruction implies a greater degree of learning than does low quality of instruction. (*b*) Success in learning can be viewed as a positive reinforcer of one's willingness to persevere on a learning task. (*c*) Therefore, though not explicitly hypothesized by Carroll, it can be logically concluded that high quality of instruction implies a greater amount of perseverance on a learning task than does low quality of instruction.

5. A high quality of instruction characterized by feedback/correction procedures did not foster a significantly greater amount of perseverance (highly controlled) among students than did a low quality of instruction characterized by the absence of feedback/correction procedures. Hence, the logical argument relative to perseverance cited in the preceding paragraph was not confirmed statistically. However, due to the highly controlled manner in which the second measure of perseverance was made, it is possible that the distinction between treatments was not severe enough to affect differentially the dependent variable. Also, it is

feasible that as quality of instruction increases with the other variables remaining constant, perseverance does not respond in a manner similar to degree of learning. This latter contention is quite feasible since perseverance more than degree of learning is assumed to be reflective of the motivational or affective aspects of the learner--an area not addressed by the operations of the model.

6. The interaction between ability to understand instruction and quality of instruction relative to a rough estimate of perseverance was of a nonsignificantly ordinal nature. The interaction between ability to understand instruction and quality of instruction relative to a highly controlled measure of perseverance was of a nonsignificantly disordinal nature. In both instances the direction of the interaction was such that students low in ability to understand instruction persevered less when subjected to a low quality of instruction than did students high in ability to understand instruction. Due largely to the failure to obtain significant main effects of ability relative to perseverance, it was not possible to confirm statistically the significant ordinal interaction between ability to understand instruction and quality of instruction relative to perseverance as logically inferred from the model and described in the following argument: (_a_) Low ability to understand instruction implies a greater need for perseverance on a learning task than does high ability to understand instruction. (_b_) High quality of instruction implies a greater amount of perseverance on a

learning task than does low quality of instruction. (c) Therefore, though not explicitly stated in the model, it can be logically inferred that the interaction between ability to understand instruction and quality of instruction relative to perseverance on a learning task is such that students low in ability to understand instruction will persevere more when subjected to high quality of instruction and less when subjected to low quality of instruction than will students high in ability to understand instruction.

7. Though not confirmed statistically, the relationship between a rough estimate of perseverance and ability to understand instruction was of an inverse nature under a high quality of instruction but of a direct nature under a low quality of instruction; furthermore, it appeared that the relationship between a rough estimate of perseverance and ability to understand instruction was not differentially affected by the treatments. Hence, although the appropriate patterns were present under the high quality of instruction and the low quality of instruction, the following logical argument was not confirmed statistically: (a) Low ability to understand instruction implies a greater need for perseverance on learning tasks than does high ability to understand instruction. (b) High quality of instruction implies a greater amount of perseverance on a learning task than does low quality of instruction. (c) Therefore, perseverance on a learning task is inversely related to ability to understand instruction under high quality of instruction but directly

related to ability to understand instruction under low quality of instruction.

8. Though not confirmed statistically, the relationship between a highly controlled measure of perseverance and ability to understand instruction was of an inverse nature under a high quality of instruction but of a direct nature under a low quality of instruction. To a significant extent, though, the relationship between a highly controlled measure of perseverance and ability to understand instruction was differentially affected by the treatments. Hence, that aspect of the following logical argument which implies differentially effective treatments was confirmed statistically: (a) Low ability to understand instruction implies a greater need for perseverance on learning tasks than does high ability to understand instruction. (b) High quality of instruction implies a greater amount of perseverance on a learning task than does low quality of instruction. (c) Therefore, perseverance on a learning task is inversely related to ability to understand instruction under high quality of instruction but directly related to ability to understand instruction under low quality of instruction.

9. Though not confirmed statistically, the relationship between degree of learning and a rough estimate of perseverance was of an inverse nature under both a high quality of instruction and a low quality of instruction. Whether this relationship was less of an inverse nature under a high quality of instruction than under a low quality of

instruction could not be confirmed statistically although the data generally supported this view. [illegible], though not confirmed statistically, the relationship between degree of learning and a highly controlled measure of perseverance was of a direct nature under a high quality of instruction but of an inverse nature under a low quality of instruction. Whether these two opposite types of relationships differed substantially from each other could not be confirmed statistically although the data generally supported this view. Hence, it was not possible to confirm statistically the exact nature of the function alluded to in Carroll's implicit suggestion that, with all other variables in the model held constant, degree of learning is a function of the amount of perseverance on a learning task.

Implications for Education

The findings and conclusions of this study have the following implications for education:

1. Further empirical verification was provided regarding Carroll's assumption that an increase in quality of instruction while the other components of the model remain constant serves to decrease the time needed by a student for learning, thus resulting in a closer approximation of mastery learning. Furthermore, the first instance of empirical support was provided regarding Carroll's hypothesized interaction between ability to understand instruction and quality of instruction relative to degree of learning. Hence,

educational administrators, instructional strategists, and curriculum developers alike have available a theoretical paradigm accompanied by supporting evidence which can serve as a basis for arriving at decisions in their respective areas.

2. Previous research has failed to address itself to the theoretical basis underlying administrative decisions regarding the organizational structuring of schools. The present study, however, served to fill this void and, in so doing, established empirical verification of a number of basic assumptions inherent in the Carroll model. Due to its heavy reliance upon the time factor in learning, the Carroll model represents an excellent basis for the theoretical justification of a form of school organization such as nongrading which has as its primary objective the provision of flexible time allotments during which a student can actively engage in a learning task until his time spent is commensurate with his time needed.

3. The employment of feedback/correction procedures in this study for the purpose of constructing a high quality of instruction served to demonstrate the efficacy of this particular instructional strategy. Though not contingent upon a nongraded or continuous-progress setting for implementation, the effectiveness of a pedagogical approach characterized by feedback/correction procedures is enhanced by a school organizational structure which provides unlimited time opportunity for learning.

4. The organizational structuring of schools in terms of a nongraded pattern as well as the construction of an instructional strategy based upon feedback/correction procedures would seem to be related very directly to decisions of a curricular nature. Of particular importance would be those decisions that foster (a) the segmenting of courses into various learning sequences and (b) the emphasizing of time as a variable and subject-matter mastery as a constant.

Recommendations for Future Research

The following recommendations are forwarded regarding future research in the area of this investigation:

1. Additional research needs to be directed toward the interaction between ability to understand instruction and quality of instruction relative to degree of learning. The present study was only the third attempt thus far to investigate Carroll's hypothesized interaction and the only one of the three which corroborated the interaction hypothesis.

2. Research is also needed regarding Carroll's hypothesized interaction when different measures of ability to understand instruction are used. Of interest would be the interaction effects of ability to understand instruction and quality of instruction relative to degree of learning when specific measures of intelligence--such as verbal ability, reading comprehension, listening skills, or some combination thereof--are employed.

3. Though not hypothesized by Carroll, the interaction between ability to understand instruction and quality of

instruction relative to perseverance should receive additional study. Despite the fact that the present investigation did not confirm statistically an interaction pattern relative to perseverance, it may be that the perseverance of a learner is significantly influenced by quality of instruction to a degree that is predictable from his ability to understand instruction.

4. The possibility of a three-way interaction among aptitude, ability to understand instruction, and quality of instruction relative to degree of learning should be investigated. Also worthy of consideration would be the three-way interaction effects of aptitude, ability to understand instruction, and quality of instruction upon perseverance as a dependent variable.

5. The three studies thus far which have investigated the various operations of the Carroll model have employed only two levels of quality of instruction. Investigations incorporating three or more treatments are needed and could possibly lead to some type of empirically-based descriptive classification of quality of instruction along a continuum from high to low.

6. Additional studies should be conducted concerning the extent to which perseverance and degree of learning are influenced similarly by the operations of Carroll's model. Of prime importance regarding research into this area would be the necessity for valid descriptions of learner behaviors indicative of perseverance as well as the accurate measurement of such behaviors.

7. The Carroll model contends that degree of learning is a function of the ratio of time spent to time needed. Investigations should be conducted to determine the exact nature of this function.

8. Studies which further explore the relationship between and among such variables as degree of learning, aptitude, ability to understand instruction, and perseverance are needed. Such investigations should address themselves to not only linear but also curvilinear relationships between and among the different variables.

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APPENDIX A

Instructional Packet for Mastery Learning Group

INSTRUCTIONAL PACKET

LEARNING SEQUENCE 332:
POLYNOMIALS AND EQUATIONS

PART I

Prerequisite: Learning Sequence 331: Notations and Open Sentences

Textbook: Dolciani, M. P., Berman, S. L., & Freilich, J. *Modern Algebra: Structure and Method--Book I*. Boston: Houghton Mifflin Company, 1962.

INTRODUCTION

This packet, which concerns itself with the first part of Learning Sequence 332: Polynomials and Equations, is made available for the purpose of helping you to identify the following:

1. The mathematical topics that you will be taught (DESCRIPTION OF CONTENT).

2. The tasks that you will be expected to perform (PERFORMANCE OBJECTIVES).

3. The learning activities that will enable you to master the necessary knowledge (ASSIGNMENTS).

4. The two trial-test stages at which your progress will be measured (FORMATIVE TRIAL TESTS I & II).

5. The final posttest stage at which your total level of achievement will be assessed (SUMMATIVE POSTTEST).

6. The instructional materials that will supplement the explanations given in the textbook (SUPPLEMENTARY EXPLANATION SHEETS).

7. The correct responses to the oral exercises that you will be assigned (ANSWERS TO ORAL EXERCISES).

DESCRIPTION OF CONTENT

The first part of this learning sequence encompasses the four basic operations of addition, subtraction, multiplication, and division with respect to directed numbers. The student is provided with instruction and practice in adding, subtracting, multiplying, and dividing two numbers of like or unlike signs. This knowledge is eventually extended and applied to mathematical problems in which more than two terms are involved.

PERFORMANCE OBJECTIVES AND CORRESPONDING ASSIGNMENTS

For the purpose of demonstrating a working knowledge of directed numbers, and after having completed the designated assignments, the student should be able to perform the following tasks in writing:

1. State the rule for finding the sum of:
    - a. Two positive numbers
    - b. Two negative numbers
    - c. A positive and a negative number.
2. Add directed numbers.

Assignments:

Read & study: pp. 116-117, 124-126; Supplementary Explanation Sheet #1, Rules for Addition of Directed Numbers.
State in writing the rule for finding the sum of two positive numbers.
State in writing the rule for finding the sum of two negative numbers.
State in writing the rule for finding the sum of a positive and a negative number.
Give three examples of each rule.
Work: p. 126, oral exercises 1-17 odd.
Work: p. 127, written exercises 1-17 odd.

3. State the rule for subtraction of directed numbers.

4. Subtract directed numbers.

Assignments:

Read & study: pp. 120-122, 128-130; Supplementary Explanation Sheet #2, Rule for Subtraction of Directed Numbers.
State in writing the rule for subtraction of directed numbers.
Work: p. 130, oral exercises 1-9 odd and 17-39 odd.
Work: p. 131, written exercises 13-19 odd.

TAKE FORMATIVE TRIAL TEST I NOW. This trial test will consist of ten problems. A score of 80 or higher will permit you to proceed to the next group of objectives and assignments. If, however, your score is below 80, then you will have to retake the trial test until at least 80 percent mastery is demonstrated. In either case, though, any test problems missed will serve to determine those assignments or "learning correctives" to which you will be recycled.

5. State the rules for multiplication of directed numbers.

6. Multiply directed numbers.

Assignments:

Read & study: pp. 133-135; Supplementary Explanation Sheet #3, Rules for Multiplication of Directed Numbers.
State in writing the rules for multiplication of directed numbers.
Work: p. 135, oral exercises 1-21 odd.
Work: p. 137, written exercises 1-19 odd. (Do not use the distributive property.)
Work: pp. 122-123, written exercises 1-11 odd.

7. State the rule for division of directed numbers.
8. Divide directed numbers.

Assignments:

Read & study: pp. 138-140; Supplementary Explanation Sheet #4, Rule for Division of Directed Numbers.
State in writing the rule for division of directed numbers.
Work: p. 140, oral exercises 1-17 odd.

TAKE FORMATIVE TRIAL TEST II NOW. This trial test will consist of ten problems. A score of 80 or higher will permit you to take the Summative Posttest which covers the entire first part of this learning sequence. If, however, your score is below 80, then you will have to retake the trial test until at least 80 percent mastery is demonstrated. In either case, though, any test problems missed will serve to determine those assignments or "learning correctives" to which you will be recycled.

TAKE THE SUMMATIVE POSTTEST NOW. The posttest will consist of 25 problems based upon the topics covered in performance objectives 1-8 and their corresponding assignments.

SUPPLEMENTARY EXPLANATION SHEET #1

RULES FOR ADDITION OF DIRECTED NUMBERS

I. Like Signs: Add the two numbers together and take the common sign for the sum.

Examples:

$$\begin{array}{r} +3 \\ +2 \\ \hline +5 \end{array} \qquad \begin{array}{r} -8 \\ -2 \\ \hline -10 \end{array}$$

$+5 + 2 = +7$

$-4 - 2 = -6$

A. Positive number + positive number = positive number

Examples:

$$\begin{array}{r} +6 \\ +7 \\ \hline +13 \end{array}$$

$+9 + 2 = +11$

B. Negative number + negative number = negative number

Examples:

$$\begin{array}{r} -5 \\ -4 \\ \hline -9 \end{array}$$

$-8 - 9 = -17$

II. Unlike Signs: Subtract the smaller from the larger and take the sign of the larger number in magnitude.

Examples:

$$\begin{array}{r} -7 \\ +4 \\ \hline -3 \end{array} \qquad \begin{array}{r} +9 \\ -3 \\ \hline +6 \end{array}$$

$-9 + 4 = -5$

$+10 - 2 = +8$

SUPPLEMENTARY EXPLANATION SHEET #2

RULE FOR SUBTRACTION OF DIRECTED NUMBERS

Change the sign of the subtrahend to its opposite and the sign of the operation to addition and follow the rules for addition.

Examples: $\begin{array}{r} +3 \\ -\ -6 \\ \hline \end{array}$ $\begin{array}{r} +3 \\ +6 \\ \hline +9 \end{array}$ $\begin{array}{r} +7 \\ -\ +4 \\ \hline \end{array}$ $\begin{array}{r} +7 \\ -4 \\ \hline +3 \end{array}$

$$+2 - (-4) = +2 + 4 = +6$$

$$-7 - (-2) = -7 + 2 = -5$$

SUPPLEMENTARY EXPLANATION SHEET #3

RULES FOR MULTIPLICATION OF DIRECTED NUMBERS

I. Like Signs: The product of two positive numbers or of two negative numbers is a positive number.

Examples: $(4)(5) = +20$

$(-6)(-3) = +18$

A. Positive number X positive number = positive number

Example: $(2)(3) = +6$

B. Negative number X negative number = positive number

Example: $(-2)(-3) = +6$

II. Unlike Signs: The product of a positive and a negative number is a negative number.

Example: $(4)(-5) = -20$

A. Positive number X negative number = negative number

Example: $(2)(-3) = -6$

B. Negative number X positive number = negative number

Example: $(-2)(3) = -6$

SUPPLEMENTARY EXPLANATION SHEET #4

RULE FOR DIVISION OF DIRECTED NUMBERS

To perform a division, replace the divisor by its reciprocal and multiply.

Examples: $$\frac{-12}{-4} = -12\left(-\frac{1}{4}\right) = 3$$

$$\frac{4}{-5} = 4\left(-\frac{1}{5}\right) = -\frac{4}{5}$$

$$\frac{-3}{5} = -3\left(\frac{1}{5}\right) = -\frac{3}{5}$$

$$50 \div (-10) = 50\left(-\frac{1}{10}\right) = -\frac{50}{10} = -5$$

$$\frac{2}{3} \div \left(-\frac{4}{5}\right) = \frac{2}{3}\left(-\frac{5}{4}\right) = -\frac{10}{12} = -\frac{5}{6}$$

ANSWERS TO ORAL EXERCISES

Page 126:

1. 13
3. 2
5. $-\frac{3}{5}$
7. 7
9. -9
11. -6.7
13. 0
15. 3
17. -9

Page 130:

1. 5
3. -8
5. 10
7. -35
9. -25
17. 22
19. 3.3
21. .08
23. -.8
25. -.2
27. -.4
29. -3.9
31. 17
33. $-\frac{1}{5}$
35. 0
37. 1
39. -1

Page 135:

1. 20
3. -21
5. 20
7. -15
9. 1
11. 1
13. $-6a^2$
15. $4b^3$
17. 0
19. -8
21. 162

Page 140:

1. 4
3. -4
5. 0
7. 1
9. $\frac{1}{3}$
11. $-\frac{1}{5}$
13. a
15. -3a
17. 5a

APPENDIX B

Instructional Packet for Control Group

INSTRUCTIONAL PACKET

LEARNING SEQUENCE 332:
POLYNOMIALS AND EQUATIONS

PART I

Prerequisite: Learning Sequence 331: Notations and Open Sentences

Textbook: Dolciani, M. P., Berman, S. L., & Freilich, J. Modern Algebra: Structure and Method--Book I. Boston: Houghton Mifflin Company, 1962.

INTRODUCTION

This packet, which concerns itself with the first part of Learning Sequence 332: Polynomials and Equations, is made available for the purpose of helping you to identify the following:

1. The mathematical topics that you will be taught (DESCRIPTION OF CONTENT).

2. The tasks that you will be expected to perform (PERFORMANCE OBJECTIVES).

3. The learning activities that will enable you to master the necessary knowledge (ASSIGNMENTS).

4. The posttest stage at which your total level of achievement will be assessed (SUMMATIVE POSTTEST).

5. The instructional materials that will supplement the explanations given in the textbook (SUPPLEMENTARY EXPLANATION SHEETS).

6. The correct responses to the oral exercises that you will be assigned (ANSWERS TO ORAL EXERCISES).

DESCRIPTION OF CONTENT

The first part of this learning sequence encompasses the four basic operations of addition, subtraction, multiplication, and division with respect to directed numbers. The student is provided with instruction and practice in adding, subtracting, multiplying, and dividing two numbers of like or unlike signs. This knowledge is eventually extended and applied to mathematical problems in which more than two terms are involved.

PERFORMANCE OBJECTIVES AND CORRESPONDING ASSIGNMENTS

For the purpose of demonstrating a working knowledge of directed numbers, and after having completed the designated assignments, the student should be able to perform the following tasks in writing:

1. State the rule for finding the sum of:
    a. Two positive numbers
    b. Two negative numbers
    c. A positive and a negative number.
2. Add directed numbers.

Assignments:

Read & study: pp. 116-117, 124-126; Supplementary Explanation Sheet #1, Rules for Addition of Directed Numbers.
State in writing the rule for finding the sum of two positive numbers.
State in writing the rule for finding the sum of two negative numbers.
State in writing the rule for finding the sum of a positive and a negative number.
Give three examples of each rule.
Work: p. 126, oral exercises 1-17 odd.
Work: p. 127, written exercises 1-17 odd.

3. State the rule for subtraction of directed numbers.

4. Subtract directed numbers.

Assignments:

Read & study: pp. 120-122, 128-130; Supplementary Explanation Sheet #2, Rule for Subtraction of Directed Numbers.
State in writing the rule for subtraction of directed numbers.
Work: p. 130, oral exercises 1-9 odd and 17-39 odd.
Work: p. 131, written exercises 13-19 odd.

5. State the rules for multiplication of directed numbers.

6. Multiply directed numbers.

Assignments:

Read & study: pp. 133-135; Supplementary Explanation Sheet #3, Rules for Multiplication of Directed Numbers.
State in writing the rules for multiplication of directed numbers.
Work: p. 135, oral exercises 1-21 odd.
Work: p. 137, written exercises 1-19 odd. (Do not use the distributive property.)
Work: pp. 122-123, written exercises 1-11 odd.

7. State the rule for division of directed numbers.

8. Divide directed numbers.

Assignments:

Read & study: pp. 138-140; Supplementary Explanation Sheet #4, Rule for Division of Directed Numbers.
State in writing the rule for division of directed numbers.
Work: p. 140, oral exercises 1-17 odd.

TAKE THE SUMMATIVE POSTTEST NOW. The posttest will consist of 25 problems based upon the topics covered in performance objectives 1-8 and their corresponding assignments.

SUPPLEMENTARY EXPLANATION SHEET #1

RULES FOR ADDITION OF DIRECTED NUMBERS

I. Like Signs: Add the two numbers together and take the common sign for the sum.

Examples:

$$\begin{array}{r} +3 \\ +2 \\ \hline +5 \end{array} \qquad \begin{array}{r} -8 \\ -2 \\ \hline -10 \end{array}$$

$+5 + 2 = +7$

$-4 - 2 = -6$

A. Positive number + positive number = positive number

Examples:

$$\begin{array}{r} +6 \\ +7 \\ \hline +13 \end{array}$$

$+9 + 2 = +11$

B. Negative number + negative number = negative number

Examples:

$$\begin{array}{r} -5 \\ -4 \\ \hline -9 \end{array}$$

$-8 - 9 = -17$

II. Unlike Signs: Subtract the smaller from the larger and take the sign of the larger number in magnitude.

Examples:

$$\begin{array}{r} -7 \\ +4 \\ \hline -3 \end{array} \qquad \begin{array}{r} +9 \\ -3 \\ \hline +6 \end{array}$$

$-9 + 4 = -5$

$+10 - 2 = +8$

SUPPLEMENTARY EXPLANATION SHEET #2

RULE FOR SUBTRACTION OF DIRECTED NUMBERS

Change the sign of the subtrahend to its opposite and the sign of the operation to addition and follow the rules for addition.

Examples:

$$\begin{array}{r} +3 \\ \underline{-\ \ -6} \end{array} \qquad \begin{array}{r} +3 \\ \underline{+6} \\ +9 \end{array} \qquad\qquad \begin{array}{r} +7 \\ \underline{-\ \ +4} \end{array} \qquad \begin{array}{r} +7 \\ \underline{-4} \\ +3 \end{array}$$

$$+2 - (-4) = +2 + 4 = +6$$

$$-7 - (-2) = -7 + 2 = -5$$

SUPPLEMENTARY EXPLANATION SHEET #3

RULES FOR MULTIPLICATION OF DIRECTED NUMBERS

I. Like Signs: The product of two positive numbers or of two negative numbers is a positive number.

Examples: (4)(5) = +20

(-6)(-3) = +18

A. Positive number X positive number = positive number

Example: (2)(3) = +6

B. Negative number X negative number = positive number

Example: (-2)(-3) = +6

II. Unlike Signs: The product of a positive and a negative number is a negative number.

Example: (4)(-5) = -20

A. Positive number X negative number = negative number

Example: (2)(-3) = -6

B. Negative number X positive number = negative number

Example: (-2)(3) = -6

SUPPLEMENTARY EXPLANATION SHEET #4

RULE FOR DIVISION OF DIRECTED NUMBERS

To perform a division, replace the divisor by its reciprocal and multiply.

Examples: $\frac{-12}{-4} = -12(-\frac{1}{4}) = 3$

$$\frac{4}{-5} = 4(-\frac{1}{5}) = -\frac{4}{5}$$

$$\frac{-3}{5} = -3(\frac{1}{5}) = -\frac{3}{5}$$

$$50 \div (-10) = 50(-\frac{1}{10}) = -\frac{50}{10} = -5$$

$$\frac{2}{3} \div (-\frac{4}{5}) = \frac{2}{3}(-\frac{5}{4}) = -\frac{10}{12} = -\frac{5}{6}$$

ANSWERS TO ORAL EXERCISES

Page 126:

1. 13
3. 2
5. $-\frac{3}{5}$
7. 7
9. -9
11. -6.7
13. 0
15. 3
17. -9

Page 130:

1. 5
3. -8
5. 10
7. -35
9. -25
17. 22
19. 3.3
21. .08
23. -.8
25. -.2
27. -.4
29. -3.9
31. 17
33. $-\frac{1}{5}$
35. 0
37. 1
39. -1

Page 135:

1. 20
3. -21
5. 20
7. -15
9. 1
11. 1
13. $-6a^2$
15. $4b^3$
17. 0
19. -8
21. 162

Page 140:

1. 4
3. -4
5. 0
7. 1
9. $\frac{1}{3}$
11. $-\frac{1}{5}$
13. a
15. -3a
17. 5a

APPENDIX C

Formative Trial Test I
(Forms A - E)

and

Answer Keys

FORMATIVE TRIAL TEST I
(FORM A)

Learning Sequence 332--Part I

_______________ (Date)

_______________ (Name)

_______________ (Period)

_______________ (Score)

PURPOSE: Formative Trial Test I is designed to measure the extent to which you have mastered the topics covered in performance objectives 1-4 and their corresponding assignments. A score of 80 or higher will permit you to proceed to the next group of objectives and assignments. If, however, your score is below 80, then you will have to retake the trial test until at least 80 percent mastery is demonstrated. In either case, though, any test problems missed will serve to determine those assignments or "learning correctives" to which you will be recycled.

DIRECTIONS: Perform the indicated operation for each of the 10 problems contained in this trial test. Please show all steps involved in computing the correct response. Also, be sure to enclose your final answer in a box "▱."

1. (Add) $\begin{array}{r} -5 \\ +2 \\ -7 \\ \underline{+3} \end{array}$

2. -8 - 4 - 2 - 3

3. 3 + (-2) + 4 + (-3)

4. y + (-8) + (-y) + 2

5. -4 + (-8 - 2)

6. (Subtract) $\begin{array}{r} -6 \\ \underline{+7} \end{array}$

7. -4 - (+5) - (-3) - (+6)

8. -(x + 6) - (-3 - x)

9. -7 - (+3 - 1) - 2

10. y - (y + 2)

ANSWER KEY

FORMATIVE TRIAL TEST I
(FORM A)

1. (Add) $\begin{array}{r} -5 \\ +2 \\ -7 \\ \underline{+3} \end{array}$

   $\boxed{-7}$

2. $-8 - 4 - 2 - 3$

   $\boxed{-17}$

3. $3 + (-2) + 4 + (-3)$

   $3 - 2 + 4 - 3$

   $7 - 5$

   $\boxed{2}$

4. $y + (-8) + (-y) + 2$

   $y - 8 - y + 2$

   $\boxed{-6}$

5. $-4 + (-8 - 2)$

   $-4 + (-10)$

   $-4 - 10$

   $\boxed{-14}$

6. (Subtract) $\begin{array}{r} -6 \\ \underline{+7} \end{array}$

   $\boxed{-13}$

7. $-4 - (+5) - (-3) - (+6)$

   $-4 - 5 + 3 - 6$

   $-15 + 3$

   $\boxed{-12}$

8. $-(x + 6) - (-3 - x)$

   $-x - 6 + 3 + x$

   $\boxed{-3}$

9. $-7 - (+3 - 1) - 2$

   $-7 - (+2) - 2$

   $-7 - 2 - 2$

   $\boxed{-11}$

10. $y - (y + 2)$

    $y - y - 2$

    $\boxed{-2}$

FORMATIVE TRIAL TEST I
(FORM B)

Learning Sequence 332--Part I

____________________ (Date)

____________________ (Name)

____________________ (Period)

____________________ (Score)

PURPOSE: Formative Trial Test I is designed to measure the extent to which you have mastered the topics covered in performance objectives 1-4 and their corresponding assignments. A score of 80 or higher will permit you to proceed to the next group of objectives and assignments. If, however, your score is below 80, then you will have to retake the trial test until at least 80 percent mastery is demonstrated. In either case, though, any test problems missed will serve to determine those assignments or "learning correctives" to which you will be recycled.

DIRECTIONS: Perform the indicated operation for each of the 10 problems contained in this trial test. Please show all steps involved in computing the correct response. Also, be sure to enclose your final answer in a box "▱."

1. (Add) $\begin{array}{r} -6 \\ +4 \\ -5 \\ \underline{+3} \end{array}$

2. $-9 - 1 - 3 - 5$

3. $4 + (-3) + 6 + (-6)$

4. $y + (-7) + (-y) + 5$

5. $-5 + (-2 - 6)$

6. (Subtract) $\begin{array}{r} -4 \\ \underline{+6} \end{array}$

7. $-9 - (+6) - (-4) - (+2)$

8. $-(x + 9) - (-4 - x)$

9. $-9 - (+4 - 3) - 4$

10. $y - (y + 3)$

ANSWER KEY

FORMATIVE TRIAL TEST I
(FORM B)

1. (Add)

$$\begin{array}{r} -6 \\ +4 \\ -5 \\ \underline{+3} \\ \boxed{-4} \end{array}$$

2. $-9 - 1 - 3 - 5$

$\boxed{-18}$

3. $4 + (-3) + 6 + (-6)$

$4 - 3 + 6 - 6$

$10 - 9$

$\boxed{1}$

4. $y + (-7) + (-y) + 5$

$y - 7 - y + 5$

$\boxed{-2}$

5. $-5 + (-2 - 6)$

$-5 + (-8)$

$-5 - 8$

$\boxed{-13}$

6. (Subtract)

$$\begin{array}{r} -4 \\ \underline{+6} \\ \boxed{-10} \end{array}$$

7. $-9 - (+6) - (-4) - (+2)$

$-9 - 6 + 4 - 2$

$-17 + 4$

$\boxed{-13}$

8. $-(x + 9) - (-4 - x)$

$-x - 9 + 4 + x$

$\boxed{-5}$

9. $-9 - (+4 - 3) - 4$

$-9 - (+1) - 4$

$-9 - 1 - 4$

$\boxed{-14}$

10. $y - (y + 3)$

$y - y - 3$

$\boxed{-3}$

FORMATIVE TRIAL TEST I
(FORM C)

Learning Sequence 332--Part I

____________________ (Date)

____________________ (Name)

____________________ (Period)

____________________ (Score)

PURPOSE: Formative Trial Test I is designed to measure the extent to which you have mastered the topics covered in performance objectives 1-4 and their corresponding assignments. A score of 80 or higher will permit you to proceed to the next group of objectives and assignments. If, however, your score is below 80, then you will have to retake the trial test until at least 80 percent mastery is demonstrated. In either case, though, any test problems missed will serve to determine those assignments or "learning correctives" to which you will be recycled.

DIRECTIONS: Perform the indicated operation for each of the 10 problems contained in this trial test. Please show all steps involved in computing the correct response. Also, be sure to enclose your final answer in a box "▱."

1. (Add) $$\begin{array}{r} -9 \\ +5 \\ -4 \\ \underline{+7} \end{array}$$

2. -7 - 3 - 4 - 2

3. 5 + (-4) + 4 + (-2)

4. y + (-6) + (-y) + 2

5. -7 + (-3 - 2)

6. (Subtract) $$\begin{array}{r} -3 \\ \underline{+9} \end{array}$$

7. -7 - (+4) - (-5) - (+1)

8. -(x + 12) - (-3 - x)

9. -6 - (+9 - 2) - 3

10. y - (y + 7)

ANSWER KEY

FORMATIVE TRIAL TEST I
(FORM C)

1. (Add)

$$\begin{array}{r} -9 \\ +5 \\ -4 \\ \underline{+7} \\ \boxed{-1} \end{array}$$

2. $-7 - 3 - 4 - 2$

$\boxed{-16}$

3. $5 + (-4) + 4 + (-2)$

$5 - 4 + 4 - 2$

$9 - 6$

$\boxed{3}$

4. $y + (-6) + (-y) + 2$

$y - 6 - y + 2$

$\boxed{-4}$

5. $-7 + (-3 - 2)$

$-7 + (-5)$

$-7 - 5$

$\boxed{-12}$

6. (Subtract)

$$\begin{array}{r} -3 \\ \underline{+9} \\ \boxed{-12} \end{array}$$

7. $-7 - (+4) - (-5) - (+1)$

$-7 - 4 + 5 - 1$

$-12 + 5$

$\boxed{-7}$

8. $-(x + 12) - (-3 - x)$

$-x - 12 + 3 + x$

$\boxed{-9}$

9. $-6 - (+9 - 2) - 3$

$-6 - (+7) - 3$

$-6 - 7 - 3$

$\boxed{-16}$

10. $y - (y + 7)$

$y - y - 7$

$\boxed{-7}$

FORMATIVE TRIAL TEST I
(FORM D)

Learning Sequence 332--Part I

______________________ (Date)	______________________ (Name)
______________________ (Period)	______________________ (Score)

PURPOSE: Formative Trial Test I is designed to measure the extent to which you have mastered the topics covered in performance objectives 1-4 and their corresponding assignments. A score of 80 or higher will permit you to proceed to the next group of objectives and assignments. If, however, your score is below 80, then you will have to retake the trial test until at least 80 percent mastery is demonstrated. In either case, though, any test problems missed will serve to determine those assignments or "learning correctives" to which you will be recycled.

DIRECTIONS: Perform the indicated operation for each of the 10 problems contained in this trial test. Please show all steps involved in computing the correct response. Also, be sure to enclose your final answer in a box "▱."

1. (Add) $\begin{array}{r} -4 \\ +1 \\ -8 \\ \underline{+5} \end{array}$

2. $-6 - 2 - 5 - 1$

3. $6 + (-3) + 5 + (-4)$

4. $y + (-9) + (-y) + 7$

5. $-9 + (-4 - 3)$

6. (Subtract) $\begin{array}{r} -4 \\ \underline{+7} \end{array}$

7. $-5 - (+1) - (-2) - (+6)$

8. $-(x + 9) - (-2 - x)$

9. $-4 - (+7 - 3) - 3$

10. $y - (y + 4)$

ANSWER KEY

FORMATIVE TRIAL TEST I
(FORM D)

1. (Add)

$$\begin{array}{r} -4 \\ +1 \\ -8 \\ \underline{+5} \\ \boxed{-6} \end{array}$$

2. $-6 - 2 - 5 - 1$

$\boxed{-14}$

3. $6 + (-3) + 5 + (-4)$

$6 - 3 + 5 - 4$

$11 - 7$

$\boxed{4}$

4. $y + (-9) + (-y) + 7$

$y - 9 - y + 7$

$\boxed{-2}$

5. $-9 + (-4 - 3)$

$-9 + (-7)$

$-9 - 7$

$\boxed{-16}$

6. (Subtract)

$$\begin{array}{r} -4 \\ \underline{+7} \\ \boxed{-11} \end{array}$$

7. $-5 - (+1) - (-2) - (+6)$

$-5 - 1 + 2 - 6$

$-12 + 2$

$\boxed{-10}$

8. $-(x + 9) - (-2 - x)$

$-x - 9 + 2 + x$

$\boxed{-7}$

9. $-4 - (+7 - 3) - 3$

$-4 - (+4) - 3$

$-4 - 4 - 3$

$\boxed{-11}$

10. $y - (y + 4)$

$y - y - 4$

$\boxed{-4}$

FORMATIVE TRIAL TEST I
(FORM E)

Learning Sequence 332--Part I

____________________ (Date)

____________________ (Name)

____________________ (Period)

____________________ (Score)

PURPOSE: Formative Trial Test I is designed to measure the extent to which you have mastered the topics covered in performance objectives 1-4 and their corresponding assignments. A score of 80 or higher will permit you to proceed to the next group of objectives and assignments. If, however, your score is below 80, then you will have to retake the trial test until at least 80 percent mastery is demonstrated. In either case, though, any test problems missed will serve to determine those assignments or "learning correctives" to which you will be recycled.

DIRECTIONS: Perform the indicated operation for each of the 10 problems contained in this trial test. Please show all steps involved in computing the correct response. Also, be sure to enclose your final answer in a box "▱."

1. (Add) $\begin{array}{r} -3 \\ +2 \\ -9 \\ \underline{+7} \end{array}$

2. $-5 - 1 - 6 - 3$

3. $7 + (-1) + 2 + (-3)$

4. $y + (-5) + (-y) + 4$

5. $-6 + (-7 - 5)$

6. (Subtract) $\begin{array}{r} -9 \\ \underline{+8} \end{array}$

7. $-2 - (+4) - (-5) - (+7)$

8. $-(x + 10) - (-4 - x)$

9. $-3 - (+8 - 4) - 4$

10. $y - (y + 8)$

ANSWER KEY

FORMATIVE TRIAL TEST I
(FORM E)

1. (Add)

$$\begin{array}{r} -3 \\ +2 \\ -9 \\ \underline{+7} \end{array}$$

$\boxed{-3}$

2. $-5 - 1 - 6 - 3$

$\boxed{-15}$

3. $7 + (-1) + 2 + (-3)$

$7 - 1 + 2 - 3$

$9 - 4$

$\boxed{5}$

4. $y + (-5) + (-y) + 4$

$y - 5 - y + 4$

$\boxed{-1}$

5. $-6 + (-7 - 5)$

$-6 + (-12)$

$-6 - 12$

$\boxed{-18}$

6. (Subtract)

$$\begin{array}{r} -9 \\ \underline{+8} \end{array}$$

$\boxed{-17}$

7. $-2 - (+4) - (-5) - (+7)$

$-2 - 4 + 5 - 7$

$-13 + 5$

$\boxed{-8}$

8. $-(x + 10) - (-4 - x)$

$-x - 10 + 4 + x$

$\boxed{-6}$

9. $-3 - (+8 - 4) - 4$

$-3 - (+4) - 4$

$-3 - 4 - 4$

$\boxed{-11}$

10. $y - (y + 8)$

$y - y - 8$

$\boxed{-8}$

APPENDIX D

Learning Correctives Based Upon Formative Trial Test I Results

LEARNING CORRECTIVES BASED UPON

FORMATIVE TRIAL TEST I RESULTS

Learning Sequence 332--Part I

____________________ (Date)

____________________ (Name)

____________________ (Period)

____________________ (Score)

☐ The score which you achieved on Formative Trial Test I was 100 ; therefore, there are no learning correctives which you must complete. Please proceed immediately to performance objectives 5-8 and their corresponding assignments.

☐ The score which you achieved on Formative Trial Test I was _____. The test items which you missed are indicated on the following pages. Please complete the designated learning correctives before proceeding to performance objectives 5-8 and their corresponding assignments.

☐ The score which you achieved on Formative Trial Test I was _____. The test items which you missed are indicated on the following pages. Please complete the designated learning correctives and then retake Formative Trial Test I. Remember, you must attain a score of 80 or higher before proceeding to the next group of objectives and assignments.

Test Item(s) Missed	Learning Correctives
☐ Any problem(s) concerning addition	☐ Review: pp. 116-117, 124-126; Supplementary Explanation Sheet #1, Rules for Addition of Directed Numbers. State in writing the rules for finding the sum of (1) two positive numbers, (2) two negative numbers, and (3) a positive and a negative number.
☐#1	☐ Work: p. 126, oral exercises 2, 4, 6, 8. Work: p. 127, written exercises 2, 6.
☐#2, ☐#3, ☐#4, or ☐#5	☐ Work: p. 126, oral exercises 16, 18. Work: p. 127, written exercise 14.
☐#2, ☐#3, ☐#4, ☐#5 (Any two or more)	☐ Work: p. 126, oral exercises 16, 17, 18. Work: p. 127, written exercises 8, 10, 12, 14.
☐ Any problem(s) concerning subtraction	☐ Review: pp. 120-122, 128-130; Supplementary Explanation Sheet #2, Rule for Subtraction of Directed Numbers. State in writing the rule for subtraction of directed numbers.
☐#6	☐ Work: p. 130, oral exercises 2, 6, 8, 10.
☐#7	☐ Work: p. 130, oral exercises 20, 36. Work: p. 131, written exercise 16.

Test Item(s) Missed	Learning Correctives
☐ #8	☐ Work: p. 130, oral exercises 37, 38. Work: p. 131, written exercises 15, 19, 20.
☐ #9	☐ Work: p. 131, written exercises 15, 17, 18.
☐ #10	☐ Work: p. 130, oral exercises 39, 40.

☐ Your most recent taking of Formative Trial Test I was your ☐ 2nd, ☐ 3rd, ☐ ___ unsuccessful attempt at obtaining a score of 80 or higher. Please consult with your teacher in order to plan for individual tutoring sessions.

ANSWERS TO ORAL EXERCISES AND EVEN-NUMBERED WRITTEN EXERCISES

Page 126, Oral Exercises:	2.	-5	16.	-1
	4.	5	17.	-9
	6.	3	18.	-3
	8.	-8		
Page 127, Even-Numbered Written Exercises:	2.	9	10.	0
	6.	32	12.	5
	8.	-2	14.	35
Page 130, Oral Exercises:	2.	8	36.	0
	6.	16	37.	1
	8.	-63	38.	1
	10.	-49	39.	-1
	20.	3	40.	-3
Page 131, Even-Numbered Written Exercises:	16.	-154		
	18.	-v		
	20.	-22		

APPENDIX E

Formative Trial Test II
(Forms A - E)

and

Answer Keys

FORMATIVE TRIAL TEST II
(FORM A)

Learning Sequence 332--Part I

____
(Date)

____
(Name)

____
(Period)

____
(Score)

PURPOSE: Formative Trial Test II is designed to measure the extent to which you have mastered the topics covered in performance objectives 5-8 and their corresponding assignments. A score of 80 or higher will permit you to take the Summative Posttest which covers the entire first part of this learning sequence. If, however, your score is below 80, then you will have to retake the trial test until at least 80 percent mastery is demonstrated. In either case, though, any test problems missed will serve to determine those assignments or "learning correctives" to which you will be recycled.

DIRECTIONS: Perform the indicated operation for each of the 10 problems contained in this trial test. Please show all steps involved in computing the correct response. Also, be sure to enclose your final answer in a box "▱."

1. $-5(2)(-3)$

2. $4(+3 + 2)$

3. $-6(-2)^3$

4. $(-\frac{1}{2})(-24)$

5. $-2(3x) + (-2)(-6)$

6. $(-30) \div (-5)$

7. $(-6a) \div 3$

8. $\frac{-35}{-7}$

9. $\frac{5}{-9}$

10. $\frac{-30x}{15}$

ANSWER KEY

FORMATIVE TRIAL TEST II
(FORM A)

1. $-5(2)(-3)$

$-10(-3)$

$\boxed{30}$

2. $4(+3 + 2)$

$4(+5)$

$\boxed{20}$

3. $-6(-2)^3$

$-6(-2)(-2)(-2)$

$12(-2)(-2)$

$-24(-2)$

$\boxed{48}$

4. $(-\frac{1}{2})(-24)$

$\boxed{12}$

5. $-2(3x) + (-2)(-6)$

$-6x + (12)$

$\boxed{-6x + 12}$

6. $(-30) \div (-5)$

$-30(-\frac{1}{5})$

$\boxed{6}$

7. $(-6a) \div 3$

$-6a(\frac{1}{3})$

$\boxed{-2a}$

8. $\frac{-35}{-7}$

$-35(-\frac{1}{7})$

$\boxed{5}$

9. $\frac{5}{-9}$

$5(-\frac{1}{9})$

$\boxed{-\frac{5}{9}}$

10. $\frac{-30x}{15}$

$-30x(\frac{1}{15})$

$\boxed{-2x}$

FORMATIVE TRIAL TEST II
(FORM B)

Learning Sequence 332--Part I

(Date)	(Name)
(Period)	(Score)

PURPOSE: Formative Trial Test II is designed to measure the extent to which you have mastered the topics covered in performance objectives 5-8 and their corresponding assignments. A score of 80 or higher will permit you to take the Summative Posttest which covers the entire first part of this learning sequence. If, however, your score is below 80, then you will have to retake the trial test until at least 80 percent mastery is demonstrated. In either case, though, any test problems missed will serve to determine those assignments or "learning correctives" to which you will be recycled.

DIRECTIONS: Perform the indicated operation for each of the 10 problems contained in this trial test. Please show all steps involved in computing the correct response. Also, be sure to enclose your final answer in a box "▱."

1. $-2(6)(-3)$

6. $(-24) \div (-8)$

2. $3(+6 + 2)$

7. $(-18a) \div 3$

3. $-5(-2)^3$

8. $\frac{+14}{-7}$

4. $(-\frac{1}{4})(-24)$

9. $\frac{7}{-9}$

5. $-5(4x) + (-5)(-6)$

10. $\frac{-25x}{5}$

ANSWER KEY

FORMATIVE TRIAL TEST II
(FORM B)

1. $-2(6)(-3)$

   $-12(-3)$

   $\boxed{36}$

2. $3(+6 + 2)$

   $3(+8)$

   $\boxed{24}$

3. $-5(-2)^3$

   $-5(-2)(-2)(-2)$

   $10(-2)(-2)$

   $-20(-2)$

   $\boxed{40}$

4. $(-\frac{1}{4})(-24)$

   $\boxed{6}$

5. $-5(4x) + (-5)(-6)$

   $-20x + (30)$

   $\boxed{-20x + 30}$

6. $(-24) \div (-8)$

   $-24(-\frac{1}{8})$

   $\boxed{3}$

7. $(-18a) \div 3$

   $-18a(\frac{1}{3})$

   $\boxed{-6a}$

8. $\frac{-14}{-7}$

   $-14(-\frac{1}{7})$

   $\boxed{2}$

9. $\frac{7}{-9}$

   $7(-\frac{1}{9})$

   $\boxed{-\frac{7}{9}}$

10. $\frac{-25x}{5}$

    $-25x(\frac{1}{5})$

    $\boxed{-5x}$

FORMATIVE TRIAL TEST II
(FORM C)

Learning Sequence 332--Part I

____________________ (Date)

____________________ (Name)

____________________ (Period)

____________________ (Score)

PURPOSE: Formative Trial Test II is designed to measure the extent to which you have mastered the topics covered in performance objectives 5-8 and their corresponding assignments. A score of 80 or higher will permit you to take the Summative Posttest which covers the entire first part of this learning sequence. If, however, your score is below 80, then you will have to retake the trial test until at least 80 percent mastery is demonstrated. In either case, though, any test problems missed will serve to determine those assignments or "learning correctives" to which you will be recycled.

DIRECTIONS: Perform the indicated operation for each of the 10 problems contained in this trial test. Please show all steps involved in computing the correct response. Also, be sure to enclose your final answer in a box "▱."

1. $-3(3)(-2)$

2. $7(+4 + 3)$

3. $-8(-2)^3$

4. $(-\frac{1}{5})(-20)$

5. $-6(5x) + (-6)(-3)$

6. $(-10) \div (-5)$

7. $(-16a) \div 2$

8. $\frac{-30}{-5}$

9. $\frac{4}{-9}$

10. $\frac{-24x}{4}$

ANSWER KEY

FORMATIVE TRIAL TEST II
(FORM C)

1. $-3(3)(-2)$

   $-9(-2)$

   $\boxed{18}$

2. $7(+4 + 3)$

   $7(+7)$

   $\boxed{49}$

3. $-8(-2)^3$

   $-8(-2)(-2)(-2)$

   $16(-2)(-2)$

   $-32(-2)$

   $\boxed{64}$

4. $(-\frac{1}{5})(-20)$

   $\boxed{4}$

5. $-6(5x) + (-6)(-3)$

   $-30x + (18)$

   $\boxed{-30x + 18}$

6. $(-10) \div (-5)$

   $-10(-\frac{1}{5})$

   $\boxed{2}$

7. $(-16a) \div 2$

   $-16a(\frac{1}{2})$

   $\boxed{-8a}$

8. $\frac{-30}{-5}$

   $-30(-\frac{1}{5})$

   $\boxed{6}$

9. $\frac{4}{-9}$

   $4(-\frac{1}{9})$

   $\boxed{-\frac{4}{9}}$

10. $\frac{-24x}{4}$

    $-24x(\frac{1}{4})$

    $\boxed{-6x}$

FORMATIVE TRIAL TEST II
(FORM D)

Learning Sequence 332--Part I

(Date)	(Name)
(Period)	(Score)

PURPOSE: Formative Trial Test II is designed to measure the extent to which you have mastered the topics covered in performance objectives 5-8 and their corresponding assignments. A score of 80 or higher will permit you to take the Summative Posttest which covers the entire first part of this learning sequence. If, however, your score is below 80, then you will have to retake the trial test until at least 80 percent mastery is demonstrated. In either case, though, any test problems missed will serve to determine those assignments or "learning correctives" to which you will be recycled.

DIRECTIONS: Perform the indicated operation for each of the 10 problems contained in this trial test. Please show all steps involved in computing the correct response. Also, be sure to enclose your final answer in a box "▱."

1. $-4(5)(-3)$

2. $2(+6 + 8)$

3. $-1(-2)^3$

4. $(-\frac{1}{6})(-18)$

5. $-7(3x) + (-7)(-2)$

6. $(-15) \div (-5)$

7. $(-28a) \div 7$

8. $\frac{-14}{-2}$

9. $\frac{2}{-9}$

10. $\frac{-32x}{4}$

ANSWER KEY

FORMATIVE TRIAL TEST II
(FORM D)

1. $-4(5)(-3)$

   $-20(-3)$

   $\boxed{60}$

2. $2(+6 + 8)$

   $2(+14)$

   $\boxed{28}$

3. $-1(-2)^3$

   $-1(-2)(-2)(-2)$

   $2(-2)(-2)$

   $-4(-2)$

   $\boxed{8}$

4. $(-\frac{1}{6})(-18)$

   $\boxed{3}$

5. $-7(3x) + (-7)(-2)$

   $-21x + (+14)$

   $\boxed{-21x + 14}$

6. $(-15) \div (-5)$

   $-15(-\frac{1}{5})$

   $\boxed{3}$

7. $(-28a) \div 7$

   $-28a(\frac{1}{7})$

   $\boxed{-4a}$

8. $\frac{-14}{-2}$

   $-14(-\frac{1}{2})$

   $\boxed{7}$

9. $\frac{2}{-9}$

   $2(-\frac{1}{9})$

   $\boxed{-\frac{2}{9}}$

10. $\frac{-32x}{4}$

    $-32x(\frac{1}{4})$

    $\boxed{-8x}$

FORMATIVE TRIAL TEST II
(FORM E)

Learning Sequence 332--Part I

(Date)	(Name)
(Period)	(Score)

PURPOSE: Formative Trial Test II is designed to measure the extent to which you have mastered the topics covered in performance objectives 5-8 and their corresponding assignments. A score of 80 or higher will permit you to take the Summative Posttest which covers the entire first part of this learning sequence. If, however, your score is below 80, then you will have to retake the trial test until at least 80 percent mastery is demonstrated. In either case, though, any test problems missed will serve to determine those assignments or "learning correctives" to which you will be recycled.

__DIRECTIONS__: Perform the indicated operation for each of the 10 problems contained in this trial test. Please show all steps involved in computing the correct response. Also, be sure to enclose your final answer in a box "▱."

1. $-8(2)(-2)$

2. $6(+2 + 6)$

3. $-7(-2)^3$

4. $(-\frac{1}{9})(-36)$

5. $-8(5x) + (-8)(-2)$

6. $(-27) \div (-3)$

7. $(-24a) \div 2$

8. $\frac{-27}{-3}$

9. $\frac{8}{-9}$

10. $\frac{-35x}{5}$

ANSWER KEY

FORMATIVE TRIAL TEST II
(FORM E)

1. $-8(2)(-2)$

   $-16(-2)$

   $\boxed{32}$

2. $6(+2+6)$

   $6(+8)$

   $\boxed{48}$

3. $-7(-2)^3$

   $-7(-2)(-2)(-2)$

   $14(-2)(-2)$

   $-28(-2)$

   $\boxed{56}$

4. $(-\frac{1}{9})(-36)$

   $\boxed{4}$

5. $-8(5x)+(-8)(-2)$

   $-40x+(16)$

   $\boxed{-40x+16}$

6. $(-27) \div (-3)$

   $-27(-\frac{1}{3})$

   $\boxed{9}$

7. $(-24a) \div 2$

   $-24a(\frac{1}{2})$

   $\boxed{-12a}$

8. $\frac{-27}{-3}$

   $-27(-\frac{1}{3})$

   $\boxed{9}$

9. $\frac{8}{-9}$

   $8(-\frac{1}{9})$

   $\boxed{-\frac{8}{9}}$

10. $\frac{-35x}{5}$

    $-35x(\frac{1}{5})$

    $\boxed{-7x}$

APPENDIX F

Learning Correctives Based Upon Formative Trial Test II Results

LEARNING CORRECTIVES BASED UPON

FORMATIVE TRIAL TEST II RESULTS

Learning Sequence 332--Part I

(Date)

(Name)

(Period)

(Score)

☐ The score which you achieved on Formative Trial Test II was 100 ; therefore, there are no learning correctives which you must complete. Please proceed immediately to the Summative Posttest.

☐ The score which you achieved on Formative Trial Test II was _____. The test items which you missed are indicated on the following pages. Please complete the designated learning correctives before proceeding to the Summative Posttest.

☐ The score which you achieved on Formative Trial Test II was _____. The test items which you missed are indicated on the following pages. Please complete the designated learning correctives and then retake Formative Trial Test II. Remember, you must attain a score of 80 or higher before proceeding to the Summative Posttest.

Test Item(s) Missed	Learning Correctives
☐ Any problem(s) concerning multiplication	☐ Review: pp. 133-135; Supplementary Explanation Sheet #3, Rules for Multiplication of Directed Numbers. State in writing the rules for multiplication of directed numbers.
☐#1	☐ Work: p. 135, oral exercises 2, 4, 7, 8, 9, 12, 14, 17.
☐#2	☐ Work: p. 137, written exercises 1, 2, 4, 6, 12, 16. (Do not use the distributive property.)
☐#3	☐ Work: p. 135, oral exercises 19, 20, 21.
☐#4	☐ Work: p. 135, oral exercises 10, 11, 16.
☐#5	☐ Work: p. 137, written exercises 7, 8, 9, 10, 11. (Do not use the distributive property.)
☐ Any problem(s) concerning division	☐ Review: pp. 138-140; Supplementary Explanation Sheet #4, Rule for Division of Directed Numbers. State in writing the rule for division of directed numbers.
☐#6 and/or ☐#7	☐ Work: p. 140, Example 4 and oral exercises 14, 15, 16, 17, 18.

Test Item(s) Missed	Learning Correctives
☐ #8	☐ Work: p. 140, Example 1 and oral exercises 1, 4, 8, 9.
☐ #9 and/or ☐ #10	☐ Work: p. 140, Examples 2 & 3 and oral exercises 2, 3, 6, 10, 11, 12.
	☐ Your most recent taking of Formative Trial Test II was your ☐ 2nd, ☐ 3rd, ☐ ___ unsuccessful attempt at obtaining a score of 80 or higher. Please consult with your teacher in order to plan for individual tutoring sessions.

ANSWERS TO ORAL EXERCISES AND EVEN-NUMBERED WRITTEN EXERCISES

Page 135, Oral Exercises:

2. 12
4. 18
7. -15
8. 28
9. 1
10. -1
11. 1
12. $-200a^2$
14. 0
16. $7d^4$
17. 0
19. -8
20. -27
21. 162

Page 137, Even-Numbered Written Exercises:

2. 21
4. -45
6. 0
8. -20
10. 0
12. -52
16. 54

Page 140, Oral Exercises:

1. 4
2. -4
3. -4
4. 4
6. 0
8. 1
9. $\frac{1}{3}$
10. -1
11. $-\frac{1}{5}$
12. -1
14. -a
15. -3a
16. -3b
17. 5a
18. 5y

APPENDIX G

Summative Posttest
(Forms I - VI)

and

Answer Keys

SUMMATIVE POSTTEST

(FORM I)

Learning Sequence 332--Part I

(Date)	(Name)
(Period)	(Score)

PURPOSE: The Summative Posttest is designed to measure the extent to which you have mastered the topics covered in the entire first part of Learning Sequence 332. More specifically, the content included in performance objectives 1-8 and their corresponding assignments is the basis for the 25 items which comprise the posttest.

DIRECTIONS: Perform the indicated operation for each of the 25 problems contained in this posttest. Please show all steps involved in computing the correct response. Also, be sure to enclose your final answer in a box "▱."

1. (Add) $\begin{array}{r} +9 \\ \underline{-5} \end{array}$

2. (Add) $\begin{array}{r} -7 \\ +5 \\ -9 \\ \underline{+6} \end{array}$

3. (Subtract) $\begin{array}{r} -7 \\ \underline{+8} \end{array}$

4. $-4 - 3 - 8 - 6$

5. $(-3) + (-6)$

6. $6 + (-4) + 8 + (-1)$

7. $y + (-6) + (-y) + 3$

8. $-6 + (-4 - 5)$

9. $(+9) - (+4)$

10. $-3 - (+2) - (-4) - (+8)$

11. $-(x + 5) - (-3 - x)$

12. $-8 - (+10 - 4) - 3$

13. $y - (y + 5)$

14. $6(-5)$

15. $5(+2 + 6)$

16. $-4(2)(-4)$

17. $3(-2)^3$

18. $(-\frac{1}{3})(-27)$

19. $-4(2x) + (-4)(-7)$

20. $(-25) \div (-5)$

21. $12 \div (-3)$

22. $(-14a) \div 2$

23. $\frac{-24}{-8}$

25. $\frac{-28x}{7}$

24. $\frac{3}{-7}$

ANSWER KEY

SUMMATIVE POSTTEST
(FORM I)

1. (Add) $\begin{array}{r} +9 \\ -5 \\ \hline \end{array}$

$\boxed{+4}$

2. (Add) $\begin{array}{r} -7 \\ +5 \\ -9 \\ +6 \\ \hline \end{array}$

$\boxed{-5}$

3. (Subtract) $\begin{array}{r} -7 \\ +8 \\ \hline \end{array}$

$\boxed{-15}$

4. $-4 - 3 - 8 - 6$

$\boxed{-21}$

5. $(-3) + (-6)$

$-3 - 6$

$\boxed{-9}$

6. $6 + (-4) + 8 + (-1)$

$6 - 4 + 8 - 1$

$14 - 5$

$\boxed{9}$

7. $y + (-6) + (-y) + 3$

$y - 6 - y + 3$

$\boxed{-3}$

8. $-6 + (-4 - 5)$

$-6 + (-9)$

$-6 - 9$

$\boxed{-15}$

9. $(+9) - (+4)$

$9 - 4$

$\boxed{5}$

10. $-3 - (+2) - (-4) - (+8)$

$-3 - 2 + 4 - 8$

$-13 + 4$

$\boxed{-9}$

11. $-(x + 5) - (-3 - x)$

$-x - 5 + 3 + x$

$\boxed{-2}$

12. $-8 - (+10 - 4) - 3$

$-8 - (+6) - 3$

$-8 - 6 - 3$

$\boxed{-17}$

13. $y - (y + 5)$

$y - y - 5$

$\boxed{-5}$

14. $6(-5)$

$\boxed{-30}$

15. $5(+2 + 6)$

$5(+8)$

$\boxed{40}$

16. $-4(2)(-4)$

$-8(-4)$

$\boxed{32}$

17. $3(-2)^3$

$3(-2)(-2)(-2)$

$-6(-2)(-2)$

$12(-2)$

$\boxed{-24}$

18. $(-\frac{1}{3})(-27)$

$\boxed{9}$

19. $-4(2x) + (-4)(-7)$

$-8x + (+28)$

$\boxed{-8x + 28}$

20. $(-25) \div (-5)$

$-25(-\frac{1}{5})$

$\boxed{5}$

21. $12 \div (-3)$

$12(-\frac{1}{3})$

$\boxed{-4}$

22. $(-14a) \div 2$

$-14a(\frac{1}{2})$

$\boxed{-7a}$

23. $\frac{-24}{-8}$

$-24(-\frac{1}{8})$

$\boxed{3}$

24. $\frac{3}{-7}$

$3(-\frac{1}{7})$

$\boxed{-\frac{3}{7}}$

25. $\frac{-28x}{7}$

$-28x(\frac{1}{7})$

$\boxed{-4x}$

SUMMATIVE POSTTEST
(FORM II)

Learning Sequence 332--Part I

____________________ (Date)

____________________ (Name)

____________________ (Period)

____________________ (Score)

PURPOSE: The Summative Posttest is designed to measure the extent to which you have mastered the topics covered in the entire first part of Learning Sequence 332. More specifically, the content included in performance objectives 1-6 and their corresponding assignments is the basis for the 25 items which comprise the posttest.

DIRECTIONS: Perform the indicated operation for each of the 25 problems contained in this posttest. Please show all steps involved in computing the correct response. Also, be sure to enclose your final answer in a box "▱."

1. (Add) $\begin{array}{r} +7 \\ \underline{-2} \end{array}$

2. (Add) $\begin{array}{r} -8 \\ +3 \\ -6 \\ \underline{+2} \end{array}$

3. (Subtract) $\begin{array}{r} -5 \\ \underline{+9} \end{array}$

4. $-3 - 5 - 7 - 4$

5. $(-4) + (-8)$

6. $5 + (-3) + 7 + (-2)$

7. $y + (-7) + (-y) + 2$

8. $-8 + (-5 - 6)$

9. $(+8) - (+5)$

10. $-2 - (+5) - (-3) - (+7)$

11. $-(x + 6) - (-2 - x)$

12. $-5 - (+9 - 3) - 4$

13. $y - (y + 6)$

14. $4(-3)$

15. $6(+3 + 4)$

16. $-3(5)(-2)$

17. $4(-2)^3$

18. $(-\frac{1}{7})(-35)$

19. $-3(4x) + (-3)(-8)$

20. $(-16) \div (-4)$

21. $14 \div (-7)$

22. $(-15a) \div 5$

23. $\frac{-24}{-6}$

25. $\frac{-21x}{7}$

24. $\frac{3}{-5}$

ANSWER KEY

SUMMATIVE POSTTEST
(FORM II)

1. (Add)
$$\begin{array}{r} +7 \\ \underline{-2} \\ \boxed{+5} \end{array}$$

2. (Add)
$$\begin{array}{r} -8 \\ +3 \\ -6 \\ \underline{+2} \\ \boxed{-9} \end{array}$$

3. (Subtract)
$$\begin{array}{r} -5 \\ \underline{+9} \\ \boxed{-14} \end{array}$$

4. $-3 - 5 - 7 - 4$

$\boxed{-19}$

5. $(-4) + (-8)$

$-4 - 8$

$\boxed{-12}$

6. $5 + (-3) + 7 + (-2)$

$5 - 3 + 7 - 2$

$12 - 5$

$\boxed{7}$

7. $y + (-7) + (-y) + 2$

$y - 7 - y + 2$

$\boxed{-5}$

8. $-8 + (-5 - 6)$

$-8 + (-11)$

$-8 - 11$

$\boxed{-19}$

9. $(+8) - (+5)$

$8 - 5$

$\boxed{3}$

10. $-2 - (+5) - (-3) - (+7)$

$-2 - 5 + 3 - 7$

$-14 + 3$

$\boxed{-11}$

11. $-(x + 6) - (-2 - x)$

$-x - 6 + 2 + x$

$\boxed{-4}$

12. $-5 - (+9 - 3) - 4$

$-5 - (+6) - 4$

$-5 - 6 - 4$

$\boxed{-15}$

13. $y - (y + 6)$

$y - y - 6$

$\boxed{-6}$

14. $4(-3)$

$\boxed{-12}$

15. $6(+3 + 4)$

$6(+7)$

$\boxed{42}$

16. $-3(5)(-2)$

$-15(-2)$

$\boxed{30}$

17. $4(-2)^3$

$4(-2)(-2)(-2)$

$-8(-2)(-2)$

$16(-2)$

$\boxed{-32}$

18. $(-\frac{1}{7})(-35)$

$\boxed{5}$

19. $-3(4x) + (-3)(-8)$

$-12x + (+24)$

$\boxed{-12x + 24}$

20. $(-16) \div (-4)$

$-16(-\frac{1}{4})$

$\boxed{4}$

21. $14 \div (-7)$

$14(-\frac{1}{7})$

$\boxed{-2}$

22. $(-15a) \div 5$

$-15a(\frac{1}{5})$

$\boxed{-3a}$

23. $\frac{-24}{-6}$

$-24(-\frac{1}{6})$

$\boxed{4}$

24. $\frac{3}{-5}$

$3(-\frac{1}{5})$

$\boxed{-\frac{3}{5}}$

25. $\frac{-21x}{7}$

$-21x(\frac{1}{7})$

$\boxed{-3x}$

SUMMATIVE POSTTEST
(FORM III)

Learning Sequence 332--Part I

(Date)	(Name)
(Period)	(Score)

PURPOSE: The Summative Posttest is designed to measure the extent to which you have mastered the topics covered in the entire first part of Learning Sequence 332. More specifically, the content included in performance objectives 1-8 and their corresponding assignments is the basis for the 25 items which comprise the posttest.

DIRECTIONS: Perform the indicated operation for each of the 25 problems contained in this posttest. Please show all steps involved in computing the correct response. Also, be sure to enclose your final answer in a box "▱."

1. $-6 + (-4 - 5)$

2. $-4 - 3 - 8 - 6$

3. $y + (-6) + (-y) + 3$

4. $(-3) + (-6)$

5. $6 + (-4) + 8 + (-1)$

6. (Subtract) $\begin{array}{r} -7 \\ \underline{+8} \end{array}$

7. (Add) $\begin{array}{r} +9 \\ \underline{-5} \end{array}$

8. (Add) $\begin{array}{r} -7 \\ +5 \\ -9 \\ \underline{+6} \end{array}$

9. $y - (y + 5)$

10. $(+9) - (+4)$

11. $-8 - (+10 - 4) - 3$

12. $-3 - (+2) - (-4) - (+8)$

13. $-(x + 5) - (-3 - x)$

14. $-4(2x) + (-4)(-7)$

15. $6(-5)$

16. $(-\frac{1}{3})(-27)$

17. $5(+2 + 6)$

18. $3(-2)^3$

19. $-4(2)(-4)$

20. $\frac{-28x}{7}$

21. $(-25) \div (-5)$

22. $\frac{3}{-7}$

23. $12 \div (-3)$

24. $\frac{-24}{-8}$

25. $(-14a) \div 2$

ANSWER KEY

SUMMATIVE POSTTEST
(FORM III)

1. -6 + (-4 - 5)

   -6 + (-9)

   -6 - 9

   $\boxed{-15}$

2. -4 - 3 - 8 - 6

   $\boxed{-21}$

3. y + (-6) + (-y) + 3

   y - 6 - y + 3

   $\boxed{-3}$

4. (-3) + (-6)

   -3 - 6

   $\boxed{-9}$

5. 6 + (-4) + 8 + (-1)

   6 - 4 + 8 - 1

   14 - 5

   $\boxed{9}$

6. (Subtract)

$$\begin{array}{r} -7 \\ \underline{+8} \\ \boxed{-15} \end{array}$$

7. (Add)

$$\begin{array}{r} +9 \\ \underline{-5} \\ \boxed{+4} \end{array}$$

8. (Add)

$$\begin{array}{r} -7 \\ +5 \\ -9 \\ \underline{+6} \\ \boxed{-5} \end{array}$$

9. y - (y + 5)

   y - y - 5

   $\boxed{-5}$

10. (+9) - (+4)

    9 - 4

    $\boxed{5}$

11. $-8 - (+10 - 4) - 3$

$-8 - (+6) - 3$

$-8 - 6 - 3$

$\boxed{-17}$

12. $-3 - (+2) - (-4) - (+8)$

$-3 - 2 + 4 - 8$

$-13 + 4$

$\boxed{-9}$

13. $-(x + 5) - (-3 - x)$

$-x - 5 + 3 + x$

$\boxed{-2}$

14. $-4(2x) + (-4)(-7)$

$-8x + (+28)$

$\boxed{-8x + 28}$

15. $6(-5)$

$\boxed{-30}$

16. $(-\frac{1}{3})(-27)$

$\boxed{9}$

17. $5(+2 + 6)$

$5(+8)$

$\boxed{40}$

18. $3(-2)^3$

$3(-2)(-2)(-2)$

$-6(-2)(-2)$

$12(-2)$

$\boxed{-24}$

19. $-4(2)(-4)$

$-8(-4)$

$\boxed{32}$

20. $\frac{-28x}{7}$

$-28x(\frac{1}{7})$

$\boxed{-4x}$

21. $(-25) \div (-5)$

$-25(-\frac{1}{5})$

$\boxed{5}$

22. $\frac{3}{-7}$

$3(-\frac{1}{7})$

$\boxed{-\frac{3}{7}}$

23. $12 \div (-3)$

$12(-\frac{1}{3})$

$\boxed{-4}$

24. $\frac{-24}{-8}$

$-24(-\frac{1}{8})$

$\boxed{3}$

25. $(-14a) \div 2$

$-14a(\frac{1}{2})$

$\boxed{-7a}$

SUMMATIVE POSTTEST
(FORM IV)

Learning Sequence 332--Part I

(Date)	(Name)
(Period)	(Score)

PURPOSE: The Summative Posttest is designed to measure the extent to which you have mastered the topics covered in the entire first part of Learning Sequence 332. More specifically, the content included in performance objectives 1-8 and their corresponding assignments is the basis for the 25 items which comprise the posttest.

DIRECTIONS: Perform the indicated operation for each of the 25 problems contained in this posttest. Please show all steps involved in computing the correct response. A[illegible] be sure to enclose your final answer in a box "▱."

1. $-8 + (-5 - 6)$

2. $-3 - 5 - 7 - 4$

3. $y + (-7) + (-y) + 2$

4. $(-a) + (-8)$

5. $5 + (-3) + 7 + (-2)$

6. (Subtract) $\begin{array}{r} -5 \\ \underline{+9} \end{array}$

7. (Add) $\begin{array}{r} +7 \\ \underline{-2} \end{array}$

8. (Add) $\begin{array}{r} -8 \\ +3 \\ -6 \\ \underline{+2} \end{array}$

9. $y - (y + 6)$

10. $(+8) - (+5)$

11. $-5 - (+9 - 3) - 4$

12. $-2 - (+5) - (-3) - (+7)$

13. $-(x + 6) - (-2 - x)$

14. $-3(4x) + (-3)(-8)$

15. $4(-3)$

16. $(-\frac{1}{7})(-35)$

17. $6(+3 + 4)$

18. $4(-2)^3$

19. $-3(5)(-2)$

20. $\frac{-21x}{7}$

21. $(-16) \div (-4)$

24. $\frac{-24}{-6}$

22. $\frac{3}{-5}$

25. $(-15a) \div 5$

23. $14 \div (-7)$

ANSWER KEY

SUMMATIVE POSTTEST
(FORM IV)

1. $-8 + (-5 - 6)$

$-8 + (-11)$

$-8 - 11$

$\boxed{-19}$

2. $-3 - 5 - 7 - 4$

$\boxed{-19}$

3. $y + (-7) + (-y) + 2$

$y - 7 - y + 2$

$\boxed{-5}$

4. $(-4) + (-8)$

$-4 - 8$

$\boxed{-12}$

5. $5 + (-3) + 7 + (-2)$

$5 - 3 + 7 - 2$

$12 - 5$

$\boxed{7}$

6. (Subtract)

$$\begin{array}{r} -5 \\ \underline{+9} \\ \boxed{-14} \end{array}$$

7. (Add)

$$\begin{array}{r} +7 \\ \underline{-2} \\ \boxed{+5} \end{array}$$

8. (Add)

$$\begin{array}{r} -8 \\ +3 \\ -6 \\ \underline{+2} \\ \boxed{-9} \end{array}$$

9. $y - (y + 6)$

$y - y - 6$

$\boxed{-6}$

10. $(+8) - (+5)$

$8 - 5$

$\boxed{3}$

11. $-5 - (+9 - 3) - 4$

$-5 - (+6) - 4$

$-5 - 6 - 4$

$\boxed{-15}$

12. $-2 - (+5) - (-3) - (+7)$

$-2 - 5 + 3 - 7$

$-14 + 3$

$\boxed{-11}$

13. $-(x + 6) - (-2 - x)$

$-x - 6 + 2 + x$

$\boxed{-4}$

14. $-3(4x) + (-3)(-8)$

$-12x + (+24)$

$\boxed{-12x + 24}$

15. $4(-3)$

$\boxed{-12}$

16. $(-\frac{1}{7})(-35)$

$\boxed{5}$

17. $6(+3 + 4)$

$6(+7)$

$\boxed{42}$

18. $4(-2)^3$

$4(-2)(-2)(-2)$

$-8(-2)(-2)$

$16(-2)$

$\boxed{-32}$

19. $-3(5)(-2)$

$-15(-2)$

$\boxed{30}$

20. $\frac{-21x}{7}$

$-21x(\frac{1}{7})$

$\boxed{-3x}$

21. $(-16) \div (-4)$

$-16(-\frac{1}{4})$

$\boxed{4}$

22. $\frac{3}{-5}$

$3(-\frac{1}{5})$

$\boxed{-\frac{3}{5}}$

23. $14 \div (-7)$

$14(-\frac{1}{7})$

$\boxed{-2}$

24. $\frac{-24}{-6}$

$-24(-\frac{1}{6})$

$\boxed{4}$

25. $(-15a) \div 5$

$-15a(\frac{1}{5})$

$\boxed{-3a}$

SUMMATIVE POSTTEST
(FORM V)

Learning Sequence 332--Part I

(Date)	(Name)
(Period)	(Score)

PURPOSE: The Summative Posttest is designed to measure the extent to which you have mastered the topics covered in the entire first part of Learning Sequence 332. More specifically, the content included in performance objectives 1-8 and their corresponding assignments is the basis for the 25 items which comprise the posttest.

DIRECTIONS: Perform the indicated operation for each of the 25 problems contained in this posttest. Please show all steps involved in computing the correct response. Also, be sure to enclose your final answer in a box "▱."

1. $6 + (-4) + 8 + (-1)$

2. $-4 - 3 - 8 - 6$

3. $-6 + (-4 - 5)$

4. $(-3) + (-6)$

5. $y + (-6) + (-y) + 3$

6. $-(x + 5) - (-3 - x)$

7. $(+9) - (+4)$

8. $y - (y + 5)$

9. $-3 - (+2) - (-4) - (+8)$

10. $-8 - (+10 - 4) - 3$

11. (Add) $\begin{array}{r} -7 \\ +5 \\ -9 \\ \underline{+6} \end{array}$

12. (Subtract) $\begin{array}{r} -7 \\ \underline{+8} \end{array}$

13. (Add) $\begin{array}{r} +9 \\ \underline{-5} \end{array}$

14. $5(+2 + 6)$

15. $3(-2)^3$

16. $-4(2x) + (-4)(-7)$

17. $-4(2)(-4)$

18. $6(-5)$

19. $(-\frac{1}{3})(-27)$

20. $12 \div (-3)$

21. $\frac{-24}{-8}$

22. $\frac{-28x}{7}$

23. $(-14a) \div 2$

25. $\frac{3}{-7}$

24. $(-25) \div (-5)$

ANSWER KEY

SUMMATIVE POSTTEST
(FORM V)

1. $6 + (-4) + 8 + (-1)$

$6 - 4 + 8 - 1$

$14 - 5$

$\boxed{9}$

2. $-4 - 3 - 8 - 6$

$\boxed{-21}$

3. $-6 + (-4 - 5)$

$-6 + (-9)$

$-6 - 9$

$\boxed{-15}$

4. $(-3) + (-6)$

$-3 - 6$

$\boxed{-9}$

5. $y + (-6) + (-y) + 3$

$y - 6 - y + 3$

$\boxed{-3}$

6. $-(x + 5) - (-3 - x)$

$-x - 5 + 3 + x$

$\boxed{-2}$

7. $(+9) - (+4)$

$9 - 4$

$\boxed{5}$

8. $y - (y + 5)$

$y - y - 5$

$\boxed{-5}$

9. $-3 - (+2) - (-4) - (+8)$

$-3 - 2 + 4 - 8$

$-13 + 4$

$\boxed{-9}$

10. $-8 - (+10 - 4) - 3$

$-8 - (+6) - 3$

$-8 - 6 - 3$

$\boxed{-17}$

11. (Add) $\begin{array}{r} -7 \\ +5 \\ -9 \\ +6 \\ \hline \end{array}$

$\boxed{-5}$

12. (Subtract) $\begin{array}{r} -7 \\ +8 \\ \hline \end{array}$

$\boxed{-15}$

13. (Add) $\begin{array}{r} +9 \\ -5 \\ \hline \end{array}$

$\boxed{+4}$

14. $5(+2 + 6)$

$5(+8)$

$\boxed{40}$

15. $3(-2)^3$

$3(-2)(-2)(-2)$

$-6(-2)(-2)$

$12(-2)$

$\boxed{-24}$

16. $-4(2x) + (-4)(-7)$

$-8x + (+28)$

$\boxed{-8x + 28}$

17. $-4(2)(-4)$

$-8(-4)$

$\boxed{32}$

18. $6(-5)$

$\boxed{-30}$

19. $(-\frac{1}{3})(-27)$

$\boxed{9}$

20. $12 \div (-3)$

$12(-\frac{1}{3})$

$\boxed{-4}$

21. $\frac{-24}{-8}$

$-24(-\frac{1}{8})$

$\boxed{3}$

22. $\frac{-28x}{7}$

$-28x(\frac{1}{7})$

$\boxed{-4x}$

23. $(-14a) \div 2$

$-14a(\frac{1}{2})$

$\boxed{-7a}$

24. $(-25) \div (-5)$

$-25(-\frac{1}{5})$

$\boxed{5}$

25. $\frac{3}{-7}$

$3(-\frac{1}{7})$

$\boxed{-\frac{3}{7}}$

SUMMATIVE POSTTEST
(FORM VI)

Learning Sequence 332--Part I

(Date)	(Name)
(Period)	(Score)

PURPOSE: The Summative Posttest is designed to measure the extent to which you have mastered the topics covered in the entire first part of Learning Sequence 332. More specifically, the content included in performance objectives 1-8 and their corresponding assignments is the basis for the 25 items which comprise the posttest.

DIRECTIONS: Perform the indicated operation for each of the 25 problems contained in this posttest. Please show all steps involved in computing the correct response. Also, be sure to enclose your final answer in a box "▱."

1. $5 + (-3) + 7 + (-2)$

2. $-3 - 5 - 7 - 4$

3. $-8 + (-5 - 6)$

4. $(-4) + (-8)$

5. $y + (-7) + (-y) + 2$

6. $-(x + 6) - (-2 - x)$

7. $(+8) - (+5)$

8. $y - (y + 6)$

9. $-2 - (+5) - (-3) - (+7)$

10. $-5 - (+9 - 3) - 4$

11. (Add) $$\begin{array}{r} -8 \\ +3 \\ -6 \\ \underline{+2} \end{array}$$

12. (Subtract) $$\begin{array}{r} -5 \\ \underline{+9} \end{array}$$

13. (Add) $$\begin{array}{r} +7 \\ \underline{-2} \end{array}$$

14. $6(+3 + 4)$

15. $4(-2)^3$

16. $-3(4x) + (-3)(-8)$

17. $-3(5)(-2)$

18. $4(-3)$

19. $(-\frac{1}{7})(-35)$

20. $14 \div (-7)$

21. $\frac{-24}{-6}$

22. $\frac{-21x}{7}$

23. $(-15a) \div 5$

25. $\frac{3}{-5}$

24. $(-16) \div (-4)$

ANSWER KEY

SUMMATIVE POSTTEST
(FORM VI)

1. $5 + (-3) + 7 + (-2)$
$5 - 3 + 7 - 2$
$12 - 5$
$\boxed{7}$

2. $-3 - 5 - 7 - 4$
$\boxed{-19}$

3. $-8 + (-5 - 6)$
$-8 + (-11)$
$-8 - 11$
$\boxed{-19}$

4. $(-4) + (-8)$
$-4 - 8$
$\boxed{-12}$

5. $y + (-7) + (-y) + 2$
$y - 7 - y + 2$
$\boxed{-5}$

6. $-(x + 6) - (-2 - x)$
$-x - 6 + 2 + x$
$\boxed{-4}$

7. $(+8) - (+5)$
$8 - 5$
$\boxed{3}$

8. $y - (y + 6)$
$y - y - 6$
$\boxed{-6}$

9. $-2 - (+5) - (-3) - (+7)$
$-2 - 5 + 3 - 7$
$-14 + 3$
$\boxed{-11}$

10. $-5 - (+9 - 3) - 4$
$-5 - (+6) - 4$
$-5 - 6 - 4$
$\boxed{-15}$

11. (Add) $\begin{array}{r} -8 \\ +3 \\ -6 \\ +2 \\ \hline \end{array}$

$\boxed{-9}$

12. (Subtract) $\begin{array}{r} -5 \\ +9 \\ \hline \end{array}$

$\boxed{-14}$

13. (Add) $\begin{array}{r} +7 \\ -2 \\ \hline \end{array}$

$\boxed{+5}$

14. $6(+3 + 4)$

$6(+7)$

$\boxed{42}$

15. $4(-2)^3$

$4(-2)(-2)(-2)$

$-8(-2)(-2)$

$16(-2)$

$\boxed{-32}$

16. $-3(4x) + (-3)(-8)$

$-12x + (+24)$

$\boxed{-12x + 24}$

17. $-3(5)(-2)$

$-15(-2)$

$\boxed{30}$

18. $4(-3)$

$\boxed{-12}$

19. $(-\frac{1}{7})(-35)$

$\boxed{5}$

20. $14 \div (-7)$

$14(-\frac{1}{7})$

$\boxed{-2}$

21. $\frac{-24}{-6}$

$-24(-\frac{1}{6})$

$\boxed{4}$

22. $\frac{-21x}{7}$

$-21x(\frac{1}{7})$

$\boxed{-3x}$

23. $(-15a) \div 5$

$-15a\left(\frac{1}{5}\right)$

$\boxed{-3a}$

24. $(-16) \div (-4)$

$-16\left(-\frac{1}{4}\right)$

$\boxed{4}$

25. $\frac{3}{-5}$

$3\left(-\frac{1}{5}\right)$

$\boxed{-\frac{3}{5}}$

APPENDIX H

Assessment of Perseverance

ASSESSMENT OF PERSEVERANCE

(Date)	(Name)

(Period)	(Time)	(Score)

PURPOSE: The first part of Learning Sequence 332, which you have just recently completed, exposed you to the operations of addition, subtraction, multiplication, and division with respect to directed numbers. In the second part of Learning Sequence 332 you will be called upon to apply your knowledge of these four basic operations to the solution of open sentences. The purpose of this exercise is twofold: (1) to introduce you to some basic concepts needed in solving open sentences by combining terms and using transformation principles and (2) to present you with a problem which involves the use of these concepts.

__DIRECTIONS__: Section I consists of a brief explanation of (1) the multiplication and division properties of equality and (2) some guidelines for combining terms and using transformation principles to solve open sentences. Section II presents one problem which involves the concepts just mentioned in solving an open sentence. Please study the instructional material and then proceed immediately to solving the problem. __Do only as much as you think you can and then notify the instructional assistant__.

SECTION I: INSTRUCTIONAL MATERIAL

I. __Multiplication Property of Equality__: For each a, each b, and each c, if $a = b$, then $a \cdot c = b \cdot c$.

__Example__: $\frac{n}{4} = 6$

This process is called __transformation by multiplication__.

$$4 \cdot \frac{n}{4} = 4 \cdot 6$$

$$n = 24$$

To "undo" a division you multiply.

II. __Division Property of Equality__: For each a, each b, and each nonzero c, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

__Example__: $6k = 84$

This process is called __transformation by division__.

$$\frac{6k}{6} = \frac{84}{6}$$

$$k = 14$$

To "undo" a multiplication you divide.

III. __Inverse Operations__: Since the operations of multiplying and dividing by the same number are opposite in effect, they are called __inverse operations__.

IV. __Guidelines for Combining Terms and Using Transformation Principles To Solve Open Sentences__:

A. Combine any similar terms in either member of the equation.

B. If there are any indicated multiplications or divisions in the variable terms, use the inverse operations to find the value of the variable. (See the two preceding examples.)

SECTION II: PROBLEM IN SOLVING AN OPEN SENTENCE

Please show all steps involved in solving the following equation:

$$\frac{(-32x \div -8) + 5(+3x - 2x + x) - 2x(4)}{\frac{1}{2}(-8 - 10 - 4) - 3(+2 + 5 - 4) + 9(3)} = \frac{5(+2 + 4) + (-2) + 8}{-3(2)(3) + 6(+7 - 4)}$$

ANSWER KEY

ASSESSMENT OF PERSEVERANCE

$$\frac{(-32x \div -8) + 5(+3x - 2x + x) - 2x(4)}{\frac{1}{2}(-8 - 10 - 4) - 3(+2 + 5 - 4) + 9(3)} = \frac{5(+2 + 4) + (-2) + 8}{-3(2)(3) + 8(+7 - 4)}$$

$$\frac{4x + 5(2x) - 8x}{\frac{1}{2}(-22) - 3(3) + 27} = \frac{5(6) - 2 + 8}{-6(3) + 8(3)}$$

$$\frac{4x + 10x - 8x}{-11 - 9 + 27} = \frac{30 - 2 + 8}{-18 + 24}$$

$$\frac{6x}{7} = \frac{36}{6}$$

$$\frac{6x}{7} = 6$$

$$7 \cdot \frac{6x}{7} = 6 \cdot 7$$

$$6x = 42$$

$$\frac{6x}{6} = \frac{42}{6}$$

$$\boxed{x = 7}$$

APPENDIX I

Record of Classes Spent by Student

RECORD OF CLASSES SPENT BY STUDENT

Learning Sequence 332--Part I

________________ (Treatment)

________________ (Name)

________________ (Period)

________________ (Student Number)

Activity	Beginning Date	Ending Date	Testing Date	Total Time
1. Performance Objectives 1-4 and Corresponding Assignments	____	____		____
2. Formative Trial Test I, Form ____			____	____
________________			____	____
________________			____	____
3. Learning Correctives Based Upon Formative Trial Test I	____	____		____
________________	____	____		____
________________	____	____		____
4. Performance Objectives 5-8 and Corresponding Assignments	____	____		____
5. Formative Trial Test II, Form ____			____	____
________________			____	____
________________			____	____
6. Learning Correctives Based Upon Formative Trial Test II	____	____		____
________________	____	____		____
________________	____	____		____
7. Summative Posttest, Form ____			____	____

APPENDIX J

Record of Student Activity and Performance

RECORD OF STUDENT ACTIVITY AND PERFORMANCE

Learning Sequence 332--Part I

____________________ (Treatment)

____________________ (Name)

____________________ (Period)

____________________ (Student Number)

Activity	Date	Performance
1. Learning Sequence 332--Part I; Began	______	N/A
2. Performance Objectives 1-4 and Corresponding Assignments; Completed	______	Satisfactory
3. Formative Trial Test I, Form ____	______	Score: ______
____________________	______	____________
____________________	______	____________
4. Learning Correctives Based Upon Formative Trial Test I; Completed	______	Satisfactory
____________________	______	____________
____________________	______	____________
5. Performance Objectives 5-8 and Corresponding Assignments; Completed	______	Satisfactory
6. Formative Trial Test II, Form ____	______	Score: ______
____________________	______	____________
____________________	______	____________
7. Learning Correctives Based Upon Formative Trial Test II; Completed	______	Satisfactory
____________________	______	____________
____________________	______	____________

Activity	Date	Performance
8. Summative Posttest, Form ___	______	Score: ________
9. Assessment of Perseverance	______	Time: ________ Score: ________

APPENDIX K

Indices of Difficulty and Discrimination for Each Item Appearing on Summative Posttest Form I

TABLE K

Indices of Difficulty and Discrimination for Each Item Appearing on Summative Posttest Form I

Item Number	Difficulty Index: Proportion Answering Item Correctly			Discrimination Index: r_{pbi} Between Item and Total Test Score		
	MLG	CG	MLG & CG	MLG	CG	MLG & CG
1	.92	.85	.89	.33	.56	.42
2	.84	.82	.83	.70	.63	.57
3	.62	.32	.48	.42	.57	.53
4	.73	.32	.54	.38	.44	.50
5	.92	.82	.87	.49	.48	.50
6	.84	.79	.82	.36	.49	.37
7	.78	.53	.66	.65	.58	.64
8	.73	.53	.63	.41	.44	.49
9	.92	.76	.85	.31	.52	.43
10	.51	.24	.38	.63	.52	.61
11	.59	.21	.41	.38	.44	.51
12	.35	.29	.32	.37	.63	.46
13	.62	.15	.39	.42	.47	.61
14	.86	.88	.87	.48	.55	.41
15	.92	.94	.93	.46	.25	.27
16	.81	.47	.65	.55	.25	.46
17	.84	.29	.58	.44	.42	.58
18	.84	.50	.68	.19	.43	.44

Item Number	Difficulty Index: Proportion Answering Item Correctly			Discrimination Index: r_{pbi} Between Item and Total Test Score		
	MLG	CG	MLG & CG	MLG	CG	MLG & CG
19	.54	.15	.35	.23	.54	.51
20	.89	.62	.76	.16	.54	.50
21	.86	.74	.80	.48	.47	.50
22	.86	.62	.75	.32	.35	.39
23	.78	.62	.70	.37	.55	.51
24	.59	.18	.39	.36	.29	.49
25	.76	.62	.69	.33	.28	.35

APPENDIX L

Indices of Difficulty and Discrimination for Each Item Appearing on Summative Posttest Form II

TABLE L

Indices of Difficulty and Discrimination for Each Item Appearing on Summative Posttest Form II

Item Number	Difficulty Index: Proportion Answering Item Correctly			Discrimination Index: r_{pbi} Between Item and Total Test Score		
	MLG	CG	MLG & CG	MLG	CG	MLG & CG
1	.89	.77	.81	.48	.54	.56
2	.81	.70	.74	.60	.48	.51
3	.70	.35	.49	.84	.64	.71
4	.70	.35	.49	.40	.33	.44
5	.93	.79	.84	.58	.45	.52
6	.85	.84	.84	.70	.34	.42
7	.89	.58	.70	.30	.33	.43
8	.56	.47	.50	.03	.17	.18
9	.78	.88	.84	.29	.08	.07
10	.56	.19	.33	.30	.51	.55
11	.67	.16	.36	.30	.44	.53
12	.48	.21	.31	.39	.69	.63
13	.56	.21	.34	.55	.49	.62
14	.89	.77	.81	.58	.54	.58
15	.85	.65	.73	.11	.51	.43
16	.89	.53	.67	.30	.43	.51
17	.74	.33	.49	.57	.46	.58
18	.78	.51	.61	.04	.45	.44

Item Number	Difficulty Index: Proportion Answering Item Correctly			Discrimination Index: r_{pbi} Between Item and Total Test Score		
	MLG	CG	MLG & CG	MLG	CG	MLG & CG
19	.44	.12	.24	.34	.51	.54
20	.89	.60	.71	.45	.38	.49
21	.89	.60	.71	.45	.51	.58
22	.81	.70	.74	.01	.38	.31
23	.89	.53	.67	.52	.51	.59
24	.67	.40	.50	.48	.47	.55
25	.70	.65	.67	.20	.48	.35

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- Phi Kappa Phi